CSE 321 Discrete Structures

Winter 2008
Lecture 13
Induction and Recursion

Announcements

- Readings
  - Monday
    - Recursion
      - 4.3 (5th Edition: 3.4)
- Midterm:
  - Friday, February 8
  - In class, closed book
  - Estimated grading weight:
    - MT 12.5%, HW 50%, Final 37.5%
- Extra Office Hour
  - Thursday, 5:30-6:20 pm, CSE 582
- Homework 5 is available

Highlights from Lecture 12

- Mathematical Induction
  - P(0)
  - $\forall$ k (P(k) $\rightarrow$ P(k+1))
  - $\therefore$ $\forall$ n P(n)
- Strong Induction
  - P(0)
  - $\forall$ k ((P(0) $\wedge$ P(1) $\wedge$ … $\wedge$ P(k)) $\rightarrow$ P(k+1))
  - $\therefore$ $\forall$ n P(n)

Induction Example

- A set of S integers is *non-divisible* if there is no pair of integers a, b in S where a divides b. If there is a pair of integers a, b in S, where a divides b, then S is *divisible.*
- Given a set S of n+1 positive integers, none exceeding 2n, show that S is divisible.
- What is the largest subset non-divisible subset of {1, 2, 3, 4, 5, 6, 7, 8, 9, 10 }.

If S is a set of n+1 positive integers, none exceeding 2n, then S is divisible

- Base case: n =1
- Suppose the result holds for n
  - If S is a set of n+1 positive integers, none exceeding 2n, then S is divisible
  - Let T be a set of n+2 positive integers, none exceeding 2n+2. Suppose T is non-divisible.

Proof by contradiction

- Claim: 2n+1 $\in$ T and 2n + 2 $\in$ T
- Claim: n+1 $\notin$ T
- Let T* = T – {2n+1, 2n+2} $\cup$ {n+1}
- If T is non-divisible, T* is also non-divisible

Recursive Definitions

- $F(0) = 0$; $F(n + 1) = F(n) + 1$;
- $F(0) = 1$; $F(n + 1) = 2 \times F(n)$;
- $F(0) = 1$; $F(n + 1) = 2^{F(n)}$

Fibonacci Numbers

- $f_0 = 0$; $f_1 = 1$; $f_n = f_{n-1} + f_{n-2}$

Bounding the Fibonacci Numbers

- Theorem: $2^{n/2} \le f_n \le 2^n$ for $n \ge 6$

Recursive Definitions of Sets

- Recursive definition
  - Basis step: $0 \in S$
  - Recursive step: if $x \in S$, then $x + 2 \in S$
  - Exclusion rule: Every element in S follows from basis steps and a finite number of recursive steps

Recursive definitions of sets

Basis: $6 \in S$; $15 \in S$;
Recursive: if $x, y \in S$, then $x + y \in S$;

Basis: $[1, 1, 0] \in S$, $[0, 1, 1] \in S$;
Recursive:
if $[x, y, z] \in S$, α in R, then $[\alpha x, \alpha y, \alpha z] \in S$
if $[x_1, y_1, z_1], [x_2, y_2, z_2] \in S$
then $[x_1 + x_2, y_1 + y_2, z_1 + z_2]$

Powers of 3

Strings

- The set Σ^* of strings over the alphabet Σ is defined
  - Basis: $\lambda \in S$ (λ is the empty string)
  - Recursive: if $w \in \Sigma^*$, $x \in \Sigma$, then $wx \in \Sigma^*$

Function definitions

$Len(\lambda) = 0;$
$Len(wx) = 1 + Len(w);$ for $w \in \Sigma^*$, $x \in \Sigma$

$Concat(w, \lambda) = w$ for $w \in \Sigma^*$
$Concat(w_1,w_2x) = Concat(w_1,w_2)x$ for w_1, w_2 in Σ^*, $x \in \Sigma$