CSE 321 Discrete Structures

Winter 2008
Lecture 16
Counting

Announcements

- Readings
  - Friday, Wednesday:
    - Counting
      - 6th edition: 5.1, 5.2, 5.3, 5th edition: 4.1, 4.2. 4.3
  - Lecture 16 video will be posted on Tuesday
  - Monday, Presidents' Day, Holiday

Highlights from Lecture 15

- Structural Induction
  - Recursive Definition
    - $\lambda \in L$
    - $w \in L$, $x \in \{a, b\}$ then $wxx \in L$
  - Recursive Function
    - $len(\lambda) = 0$
    - $w \in \Sigma^*$, $x \in \Sigma$, $len(wx) = 1 + len(w)$
  - Prove all words in L have even length

Counting

- Determining the number of elements of a finite set

Counting Rules

Product Rule: If there are n_1 choices for the first item and n_2 choices for the second item, then there are $n_1 n_2$ choices for the two items

Sum Rules: If there are n_1 choices of an element from S_1 and n_2 choices of an element from S_2 and $S_1 \cap S_2$ is empty, then there are $n_1 + n_2$ choices of an element from $S_1 \cup S_2$

Counting examples

License numbers have the form LLL DDD, how many different license numbers are available?

There are 38 students in a class, and 38 chairs, how many different seating arrangements are there if everyone shows up?

How many different predicates are there on $\Sigma = \{a,\ldots,z\}$?

Important cases of the Product Rule

- Cartesian product
  - $|A_1 \times A_2 \times \ldots \times A_n| = |A_1||A_2|. . . |A_n|$
- Subsets of a set S
  - $|\boldsymbol{P}(S)| = 2^{|S|}$
- Strings of length n over Σ
  - $|\Sigma^n| = |\Sigma|^n$

Counting Functions

Suppose |S| = n, |T| = m

How many functions from S to T?

How many one-to-one functions from S to T?

More complicated counting examples

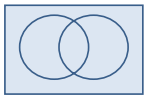
- BASIC variable names
  - Variables can be one or two characters long
    - The first character must be a letter
    - The second character can be a letter or a digit
    - The keywords “TO”, “IF”, and “DO” are excluded

Counting Passwords

- Passwords must be 4 to 6 characters long, and must contain at least on letter and at least one digit. (Case insensitive, no special characters)

Inclusion-Exclusion Principle

$|A_1 \cup A_2| = |A_1| + |A_2| - |A_1 \cap A_2|$

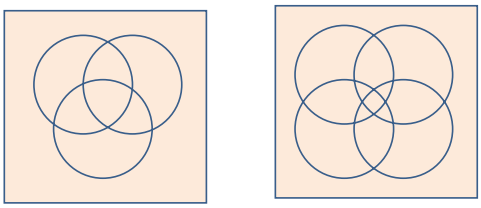


- How many strings of length 9 start with 00 or end with 11

Inclusion-Exclusion

- A class has of 40 students has 20 CS majors, 15 Math majors. 5 of these students are dual majors. How many students in the class are neither math, nor CS majors?

Generalizing Inclusion Exclusion



Pigeon Hole Principle

If k is a positive integer and k+1 or more objects are placed into k boxes, then at least one box has two or more objects

If N objects are placed into k boxes, then there is at least one box containing at least $\lceil N/k \rceil$ objects

PHP Applications

- Prove that if a city has at least 10 million phone subscribers it needs more than one area code. (Phone numbers of the form NXX-XXXX.)
- Prove that if you have 800 people, at least three share a common birthday.

Clever PHP Applications

- Every sequence of $n^2 + 1$ distinct numbers contains a subsequence of length n+1 that is either strictly increasing or strictly decreasing.

  4, 22, 8, 15, 19, 11, 2, 1, 9, 20, 10, 7, 16, 3, 6, 5, 14

Proof

- Let $a_1, \ldots a_m$ be a sequence of n^2+1 distinct numbers
- Let i_k be the length of the longest increasing sequence starting at a_k
- Let d_k be the length of the longest decreasing sequence starting at a_k
- Suppose $i_k \leq n$ and $d_k \leq n$ for all k
- There must be k and j, k < j, with $i_k = i_j$ and $d_k = d_j$