# CSE 321  Discrete Structures

Winter 2008
Lecture 18
Generalized Permutations and
Counting

# Announcements

- Readings
  - Counting
    - 5.5, (4.5) Generalized Permutations and Combinations
  - Probability Theory
    - 6.1, 6.2 (5.1, 5.2) Probability Theory
    - 6.3 (New material!) Bayes' Theorem
    - 6.4 (5.3) Expectation
  - Advanced Counting Techniques – Ch 7.
    - Not covered

# Highlights from Lecture 17

- Permutations

$$P(n, r) = n(n-1)(n-2)\cdots(n-r+1) = \frac{n!}{(n-r)!}$$

- Combinations

$$C(n, r) = \binom{n}{r} = \frac{n!}{(n-r)!r!}$$

# How many

- Let $s_1$ be a string of length n over $\Sigma_1$
- Let $s_2$ be a string of length m over $\Sigma_2$
- Assuming $\Sigma_1$ and $\Sigma_2$ are distinct, how many interleavings are there of $s_1$ and $s_2$?

# Permutations with repetition

# Combinations with repetition

- How many different ways are there of selecting 5 letters from {A, B, C} with repetition

How many non-decreasing sequences of {1,2,3} of length 5 are there?

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How many different ways are there of adding 3 non-negative integers together to get 5 ?

1 + 2 + 2        • | • • | • •

2 + 0 + 3        • • | | • • •

0 + 1 + 4

3 + 1 + 1

5 + 0 + 0

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C(n+r-1,n-1) r-combinations of an n element set with repetition

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# Permutations of indistinguishable objects

- How many different strings can be made from reordering the letters ABCDEFGH

- How many different strings can be made from reordering the letters AAAABBBB

- How many different strings can be made from reordering the letters GOOOOGLE

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# Discrete Probability

**Experiment**: Procedure that yields an outcome

**Sample space**: Set of all possible outcomes

**Event**: subset of the sample space

S a sample space of equally likely outcomes, E an event, the probability of E, $p(E) = |E|/|S|$

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# Example: Dice

## Example: Poker

Probability of 4 of a kind

## Combinations of Events

$E^C$ is the complement of E

$P(E^C) = 1 - P(E)$

$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$