CSE 321 Discrete Structures

Winter 2008
Lecture 21
Probability: Expectation, Analysis of Algorithms

Announcements

- Readings
  - Probability Theory
    - 6.4 (5.3) Expectation
  - Advanced Counting Techniques – Ch 7.
    - Not covered
  - Relations
    - Chapter 8 (Chapter 7)

Highlights from Lecture 20: Bayes' Theorem

Suppose that E and F are events from a sample space S such that p(E) > 0 and p(F) > 0. Then

$$p(F \mid E) = \frac{p(E \mid F)p(F)}{p(E \mid F)p(F) + p(E \mid \overline{F})p(\overline{F})}$$

Testing for disease

Disease is very rare: p(D) = 1/100,000

Testing is accurate:
False negative: 1%
False positive: 0.5%

Suppose you get a positive result, what do you conclude?

p(D | Y) ≅ 1/500

Expectation

The expected value of random variable X(s) on sample space S is:

$$E(X) = \sum_{x \in S} p(s)X(s)$$

$$E(X) = \sum_{r \in X(S)} p(X = r)r$$

Flip a coin until the first head Expected number of flips?

Probability Space:

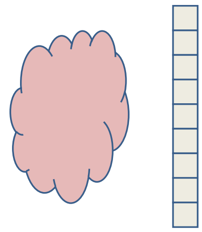
Computing the expectation:

Linearity of Expectation

$E(X_1 + X_2) = E(X_1) + E(X_2)$

$E(aX) = aE(X)$

Hashing



$H: M \rightarrow [0..n-1]$

If k elements have been hashed to random locations, what is the expected number of elements in bucket j?

What is the expected number of collisions when hashing k elements to random locations?

Hashing analysis

Sample space: $[0..n-1] \times [0..n-1] \times . . . \times [0..n-1]$

Random Variables
X_j = number of elements hashed to bucket j
C = total number of collisions

B_{ij} = 1 if element i hashed to bucket j
B_{ij} = 0 if element i is not hashed to bucket j

C_{ab} = 1 if element a is hashed to the same bucket as element b
C_{ab} = 0 if element a is hashed to a different bucket than element b

Counting inversions

Let $p_1, p_2, . . . , p_n$ be a permutation of 1 . . . n
p_i, p_j is an inversion if $i < j$ and $p_i > p_j$

4, 2, 5, 1, 3

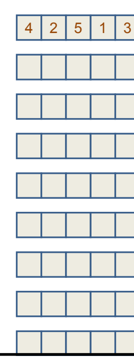
1, 6, 4, 3, 2, 5

7, 6, 5, 4, 3, 2, 1

Expected number of inversions for a random permutation

Insertion sort

```
for i :=1 to n-1
     j := i;
     while j > 0 and A[ j - 1 ] > A[ j ]
           swap(A[ j -1], A[ j ]);
```



Expected number of swaps for Insertion Sort

Left to right maxima

```
max_so_far := A[0];
for i := 1 to n-1
     if (A[ i ] > max_so_far)
         max_so_far := A[ i ];
```

5, 16, 9, 14, 11, 18, 7, 2, 1, 20, 3, 19, 10, 15, 4, 6, 17, 18, 8

What is the expected number of left-to-right maxima in a random permutation