$\neg p \vee q$

"implication"

$p \leftrightarrow q$	p	q	$p \to q$	$\neg p \vee q$	$\neg q$	$\neg p$	$\neg q \to \neg p$
T	F	F	T	T	T	T	T
F	F	T	T	T	F	T	T
F	T	F	F	F	T	F	F
T	T	T	T	T	F	F	T

defn

$q \to p$ converse

$\neg q \to \neg p$ contrapositive

dog → mammal

mammal → dog ← converse

¬mammal → ¬dog ← contrapositive

Two (compound) propositions a, b are **logically equivalent** if they have the same truth table.

$a \equiv b$

a & b are logically equiv means $a \leftrightarrow b$ is a tautology

$$p \rightarrow q \equiv \neg q \rightarrow \neg p$$

$$p \rightarrow q \equiv \neg p \vee q$$

TABLE 6 Logical Equivalences.

Equivalence	*Name*
$p \wedge \mathbf{T} \equiv p$ $p \vee \mathbf{F} \equiv p$	Identity laws
$p \vee \mathbf{T} \equiv \mathbf{T}$ $p \wedge \mathbf{F} \equiv \mathbf{F}$	Domination laws
$p \vee p \equiv p$ $p \wedge p \equiv p$	Idempotent laws
$\neg(\neg p) \equiv p$	Double negation law
$p \vee q \equiv q \vee p$ $p \wedge q \equiv q \wedge p$	Commutative laws
$(p \vee q) \vee r \equiv p \vee (q \vee r)$ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	Associative laws
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	Distributive laws
$\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$	De Morgan's laws
$p \vee (p \wedge q) \equiv p$ $p \wedge (p \vee q) \equiv p$	Absorption laws
$p \vee \neg p \equiv \mathbf{T}$ $p \wedge \neg p \equiv \mathbf{F}$	Negation laws

$$p \to q \equiv \neg p \vee q$$
$$\equiv q \vee \neg p \quad \text{(commutative)}$$
$$\equiv \neg(\neg q) \vee \neg p \quad \text{(double neg)}$$
$$\equiv \neg q \to \neg p$$

an implication is log. equiv. to its contrapositive

$$p \to (p \vee q) \equiv \neg p \vee (p \vee q)$$
$$\equiv (\neg p \vee p) \vee q$$
$$\equiv (p \vee \neg p) \vee q$$
$$\equiv T \vee q$$
$$\equiv q \vee T$$
$$\equiv T$$

Propositional function

Predicate

$E(x)$

Universe = Integers or
aka
Domain
= Reals or
= Animals or

...

$E(x)$ is true if x is an even integer

$C(x)$ is true if x is composite

e_3	F	c_3	F	$(e_3 \to c_3)$
e_4	T	c_4	T	$\wedge (e_4 \to c_4)$
e_5	F	c_5	F	$\wedge (e_5 \to c_5)$
$\vdots$	T	c_6	T	$\wedge \ \vdots$
	$\vdots$			

all even ints > 2 are composite

$\forall x \quad E(x) \to C(x)$