TABLE 1 Rules of Inference.

Rule of Inference	*Tautology*	*Name*
p $p \to q$ $\therefore q$	$[p \wedge (p \to q)] \to q$	Modus ponens
$\neg q$ $p \to q$ $\therefore \neg p$	$[\neg q \wedge (p \to q)] \to \neg p$	Modus tollens
$p \to q$ $q \to r$ $\therefore p \to r$	$[(p \to q) \wedge (q \to r)] \to (p \to r)$	Hypothetical syl Syllogism
$p \vee q$ $\neg p$ $\therefore q$	$[(p \vee q) \wedge \neg p] \to q$	Disjunctive syllo Syllogism
p $\therefore p \vee q$	$p \to (p \vee q)$	Addition
$p \wedge q$ $\therefore p$	$(p \wedge q) \to p$	Simplification
p q $\therefore p \wedge q$	$[(p) \wedge (q)] \to (p \wedge q)$	Conjunction
$p \vee q$ $\neg p \vee r$ $\therefore q \vee r$	$[(p \vee q) \wedge (\neg p \vee r)] \to (q \vee r)$	Resolution

h_1
h_2
h_3
$\therefore c$

$(h_1 \wedge h_2 \wedge h_3) \to c$

Should be tautology

TABLE 2 **Rules of Inference for Quantified Statements.**

Rule of Inference	*Name*
$\dfrac{\forall x P(x)}{\therefore P(c)}$	Universal instantiation
$\dfrac{P(c) \text{ for an arbitrary } c}{\therefore \forall x P(x)}$	Universal generalization
$\dfrac{\exists x P(x)}{\therefore P(c) \text{ for some element } c}$	Existential instantiation
$\dfrac{P(c) \text{ for some element } c}{\therefore \exists x P(x)}$	Existential generalization

(a) all lions are fierce
(b) Some lions don't drink coffee
(c) Some fierce animals don't drink coffee

$F(x)$ x is fierce
$L(x)$ x is a lion
$C(x)$ x drinks coffee

(a) $\forall x\,(L(x) \to F(x))$
(b) $\exists x\,(L(x) \wedge \neg C(x))$
(c) $\exists x\,(F(x) \wedge \neg C(x))$

(b) + existential instantiation

there is some x, say x_0
st $L(x_0) \wedge \neg C(x_0)$ is true

(a) $\forall x\ L(x) \to F(x)$

$L(x_0) \to F(x_0)$

+ simp.

$L(x_0)$

$\therefore\ F(x_0)$ by m.p.

+ simp.
+ conjunct.

$\neg C(x_0)$

$F(x_0) \wedge \neg C(x_0)$ conjunction

$\exists x\ F(x) \wedge \neg C(x)$

Proofs

$p \rightarrow q$

direct proof

Ex1: If n is odd, n^2 is odd

$n = 2k+1$

$n^2 = (2k+1)^2 = 4k^2+4k+1$

$= 2(2k^2+2k)+1$

Ex2?: if $3n+2$ is odd, then n is odd

$3n+2 = 2k+1$

$n = \frac{2k+1-2}{3}$

Dead end

Proof by contra-position

if n is even then $3n+2$ is even

$n = 2k \rightarrow 3n+2 = 3(2k)+2$

$= 2(3k+1)$, $\therefore$ even

$P(n)$: if $n>1$ then $n^2>n$

$P(0)$: $\underbrace{\text{if } 0>1 \text{ then } \ldots\ldots}_{\text{True}}$ doesn't come

<u>Vacuous proof</u>

$\forall n\, P(n)$

$P(2)$ is trivially true

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if $R(n)$ then $Q(n)$

$\forall n \quad R(n) \rightarrow Q(n)$

if $r \equiv F$ then $r \rightarrow q$ is T, regardless of q

eg: $R(n) \equiv n = 42$

$Q(n)$ is true

$R(n) \rightarrow Q(n)$ is true, regardless of R

<u>trivial</u> proof

proof by contradiction

prove p

$\neg p \rightarrow \underline{F}$

$\sqrt{2}$ is irrational

assume

$\sqrt{2} = a/b$ in lowest terms (ie, a, b ints with no common factors)

$2 = a^2/b^2$

$2b^2 = a^2$

a^2 is even, a is even

$a = 2K$

$2b^2 = (2K)^2 = 4K^2$

$b^2 = 2K^2$

b is even

2 is common factor

TILT