

TABLE 1 Rules of Inference.

Rule of Inference	Tautology	Name
$\begin{array}{l} p \\ p \rightarrow q \\ \hline \therefore q \end{array}$	$[p \wedge (p \rightarrow q)] \rightarrow q$	Modus ponens
$\begin{array}{l} \neg q \\ p \rightarrow q \\ \hline \therefore \neg p \end{array}$	$[\neg q \wedge (p \rightarrow q)] \rightarrow \neg p$	Modus tollens
$\begin{array}{l} p \rightarrow q \\ q \rightarrow r \\ \hline \therefore p \rightarrow r \end{array}$	$[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$	Hypothetical syllogism
$\begin{array}{l} p \vee q \\ \neg p \\ \hline \therefore q \end{array}$	$[(p \vee q) \wedge \neg p] \rightarrow q$	Disjunctive syllogism
$\begin{array}{l} p \\ \hline \therefore p \vee q \end{array}$	$p \rightarrow (p \vee q)$	Addition
$\begin{array}{l} p \wedge q \\ \hline \therefore p \end{array}$	$(p \wedge q) \rightarrow p$	Simplification
$\begin{array}{l} p \\ q \\ \hline \therefore p \wedge q \end{array}$	$[(p) \wedge (q)] \rightarrow (p \wedge q)$	Conjunction
$\begin{array}{l} p \vee q \\ \neg p \vee r \\ \hline \therefore q \vee r \end{array}$	$[(p \vee q) \wedge (\neg p \vee r)] \rightarrow (q \vee r)$	Resolution

$$\begin{array}{l} h_1 \\ h_2 \\ h_3 \\ \hline \therefore C \end{array}$$

$$(h_1 \wedge h_2 \wedge h_3) \rightarrow C$$

Should be tautology

TABLE 2 Rules of Inference for Quantified Statements.

<i>Rule of Inference</i>	<i>Name</i>
$\frac{\forall x P(x)}{\therefore P(c)}$	Universal instantiation
$\frac{P(c) \text{ for an arbitrary } c}{\therefore \forall x P(x)}$	Universal generalization
$\frac{\exists x P(x)}{\therefore P(c) \text{ for some element } c}$	Existential instantiation
$\frac{P(c) \text{ for some element } c}{\therefore \exists x P(x)}$	Existential generalization

- (a) all lions are fierce
 (b) Some lions don't drink coffee
 (c) Some fierce animals don't drink coffee

$F(x)$ x is fierce

$L(x)$ x is a lion

$C(x)$ x drinks coffee

(a) $\forall x (L(x) \rightarrow F(x))$

(b) $\exists x (L(x) \wedge \neg C(x))$

(c) $\exists x (F(x) \wedge \neg C(x))$

(b) + existential instantiation

there is some x , say x_0

st $L(x_0) \wedge \neg C(x_0)$ is true

(a) $\forall x L(x) \rightarrow F(x)$

$L(x_0) \rightarrow F(x_0)$

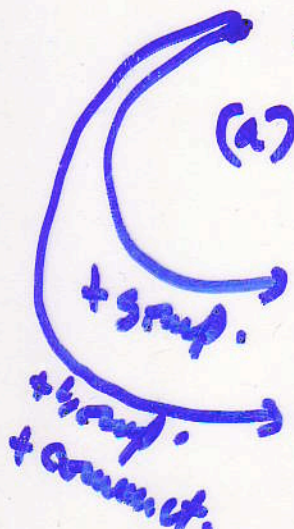
$L(x_0)$

$\therefore F(x_0)$ by m.p.

$\neg C(x_0)$

$F(x_0) \wedge \neg C(x_0)$

conjunction



Proofs

$$P \rightarrow Q$$

direct proof

EX1: if n is odd, n^2 is odd

$$n = 2k + 1$$

$$n^2 = (2k + 1)^2 = 4k^2 + 4k + 1$$

$$= 2(2k^2 + 2k) + 1$$

EX2?: if $3n + 2$ is odd, then n is odd

$$3n + 2 = 2k + 1$$

$$n = \frac{2k + 1 - 2}{3}$$

Dead end

Proof by
Contra-
position

if n is even then $3n + 2$ is even

$$n = 2k \rightarrow 3n + 2 = 3(2k) + 2$$

$$= 2(3k + 1), \therefore \text{even}$$

$P(n)$: if $n > 1$ then $n^2 > n$

$P(0)$: if $0 > 1$ then \dots ^{double check} True

Vacuous proof

$\forall n P(n)$

$P(2)$ is trivially true

if $R(n)$ then $Q(n)$

$\forall n R(n) \rightarrow Q(n)$

eg: $R(n) \equiv n = 42$

if $r \equiv F$
then
 $r \rightarrow q$ is T,
regardless of q

$Q(n)$ is true

$R(n) \rightarrow Q(n)$ is true.

trivial proof

regardless of R

proof by contradiction

prove P

$$\neg P \rightarrow \underline{\underline{F}}$$

$\sqrt{2}$ is irrational

assume

$$\sqrt{2} = \frac{a}{b}$$

in lowest terms
(ie, a, b ints with
no common
factors) ←

$$2 = \frac{a^2}{b^2}$$

$$2b^2 = a^2$$

a^2 is even, a is even

$$a = 2k$$

$$2b^2 = (2k)^2 = 4k^2$$

$$b^2 = 2k^2$$

b is even

2 is common factor ←