Deductive Inference

Rigorous

Inference of particular instances from general laws

Inductive Inference

useful but NOT Rigorous

inference of general laws from particular instances

Mathematical induction

rigorous

Way to prove a pattern

FALSE

$*_1$ Claim $\forall$ natural numbers n, n is uniquely expressible as a minimal length the sum of cubes

n	n^3
0	0
1	1
2	8
3	27
4	64
$\vdots$	

$0 = 0^3$

$1 = 1^3$

$2 = 1^3 + 1^3$

$\vdots$

$7 = 1^3 + 1^3 + \cdots + 1^3$

$8 = 2^3$

$9 = 2^3 + 1^3$

$10 = 2^3 + 1^3 + 1^3$

$15 = 2^3 + 7 \times 1^3$

$16 = 2^3 + 2^3$

$\vdots$

$26 = 2 \times 2^3 + 10 \times 1^3$

$27 = 3^3$

$\vdots$

$1729 = 12^3 + 1^3 = 10^3 + 9^3$

Counterexample!

Mathematical Induction

Propositional Function $P(n)$
(n a positive integer)

$$\Big[\underbrace{P(1)}_{\text{basis}} \wedge \forall k\,(\underbrace{P(k) \to P(k+1)}_{\text{inductive hypothesis}})\Big] \to \forall n\, P(n)$$

P(n): any $2^n \times 2^n$ board missing any one square can be tiled with 'L's.

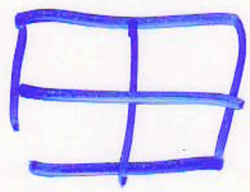
$P(n) \to P(n+1)$

allows hypothetical tilings of $2^n \times 2^n$ boards to be used to build $2^{n+1} \times 2^{n+1}$ tiling

Basis

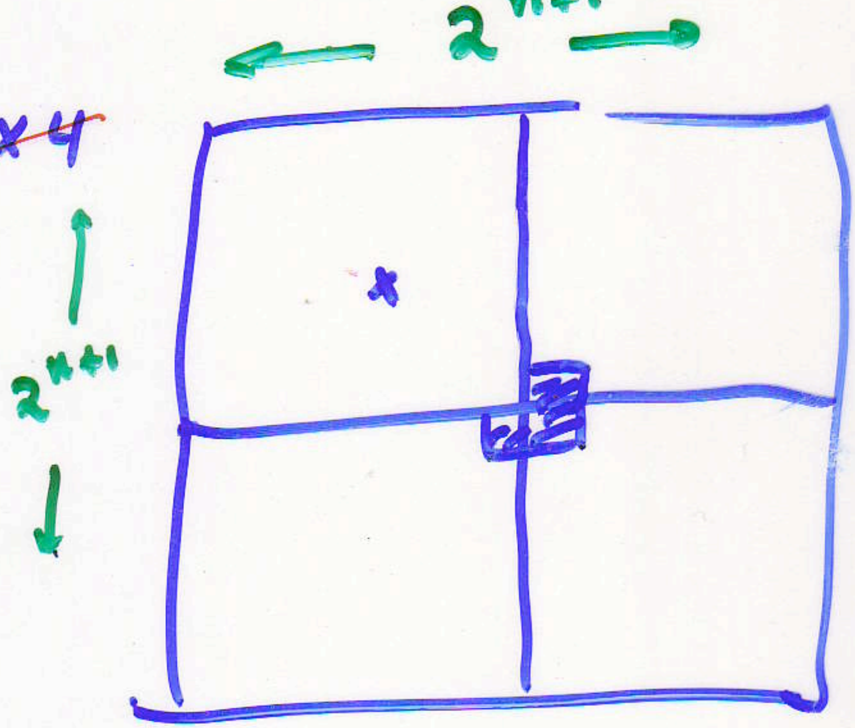
$n=0$

$n=1$



~~$n=2$~~ ~~4×4~~

$2^{\cancel{k}\, n+1} \times 2^{\cancel{k}\, n+1}$



ind. step

$\underline{\forall n}\ P(n)$

7-4 & 6-5

$2^1-1=1 = \overset{2^0}{1}$

$2^2-1=3 = 1 + \overset{2^1}{2}$

$2^3-1=7 = 1 + 2 + \overset{2^2}{4}$

$2^4-1=15 = 1 + 2 + 4 + \underset{2^3}{8}$

$$\left[2^n-1 = 1+2+2^2+2^3 \dots 2^{n-1}\right] \quad P(n)$$

Basis

$P(1) = 2^1 - 1 \equiv 1$ true

Induction

assume $P(k)$ $k \geq 1$

to show $P(k+1)$ is true

$P(k+1)$

$$\underbrace{1+2+2^2+\dots 2^{k-1}}_{2^k-1} + 2^k \quad \text{by ind. hyp.}$$

$$+2^k$$

$$= 2 \cdot 2^k - 1 \qquad \therefore P(k) \rightarrow P(k+1)$$

$$= 2^{k+1} - 1 \qquad \text{so by ind. } \forall n P(n)$$