Sets

unordered collection of objects

$A = \{0, 2, 4, 6, 8\}$

$|A| = 5$

$\{x \mid P(x)\}$

$B = \{x \mid x$ is an even nonnegative integer $\}$

$C = \{2, 8, 4, 6, 0, 0, 2\}$

$|C| = 5$

all same set.

Two sets A & B are equal if

for all $x \in A \leftrightarrow x \in B$

member of
element of

universe

$\{ x \in U \mid P(x) \}$

$\emptyset = \{ \}$ null set empty set

$\phi \neq \{ \phi \} \neq \{ \{ \phi \} \}$

cardinality $|A|$

"# of elements"

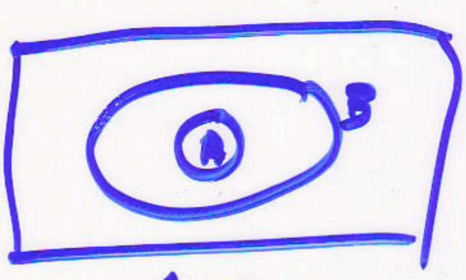
$|\phi| = 0$

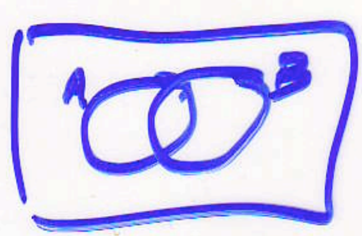
$|\{\phi\}| = 1$

Subset $A \subseteq B$

if $x \in A$ then $x \in B$

U





$A \subseteq B$

$A \nsubseteq B$

$A \subseteq A \quad \forall A$

$A \subset B$ subset but not $=$

$A \subsetneq B$ proper subset

$\phi \subseteq A$ for all A

$\phi \subset A$ if A is not empty

$\mathcal{P}(A)$ power set of A

$= \{ B \mid B \subseteq A \}$

$A = \{1, 2, 3\}$

no 3

$$\mathcal{P}(A) = \{ \phi, \{1\}, \{2\}, \{3\}, \{2, 1\}, \{1, 3\}, \{3, 2\}, \{1, 2, 3\} \}$$

have 3

$|\mathcal{P}(A)| = 8$

$$A \cap B = \{x \mid x \in A \wedge x \in B\}$$

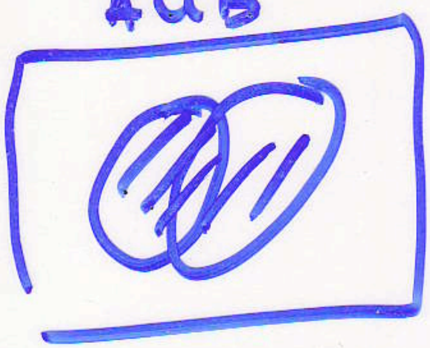
$$A \cup B = \{x \mid x \in A \vee x \in B\}$$

$$|A \cup B| = |A| + |B| - |A \cap B|$$

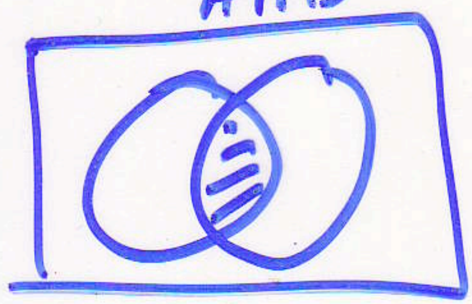
$A \cap B = \phi$ A and B are <u>disjoint</u>

$$A - B = \{x \mid x \in A \text{ and } x \notin B\}$$

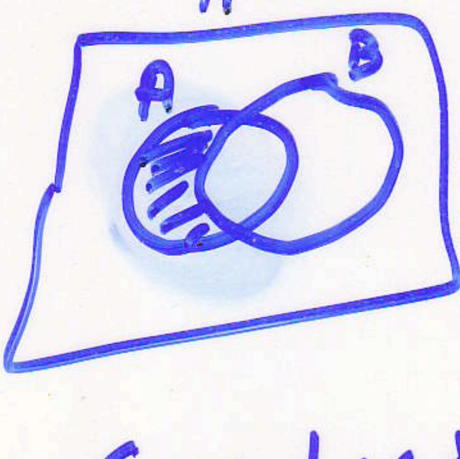
$A \cup B$

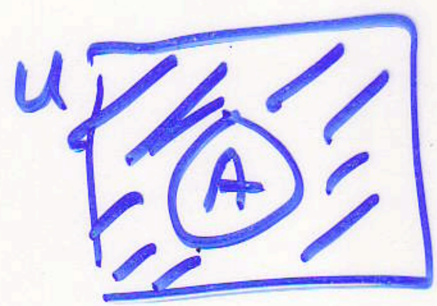


$A \cap B$



$A - B$





$$\overline{A} = \{x \mid x \notin A\}$$

Fact

$$|\mathcal{P}(A)| = 2^{|A|} \quad (*)$$

Proof: by induction on $|A|$

Basis

ϕ, $|\phi| = 0$

$\mathcal{P}(\phi) = \{\phi\}$

$|\mathcal{P}(\phi)| = 1 \qquad 2^0 = 1$ ✓

ind

Suppose $(*)$ holds for all sets of cardinality $\leq n$

$|A| = n+1 > 0$

let $a \in A$

B = set A excluding element a

$|B| = n$

By Ind $|\mathcal{P}(B)| = 2^n$

$\mathcal{P}(A) = \{C \in \mathcal{P}(B)\} \cup \{C \cup \{a\} \mid C \in \mathcal{P}(B)\}$

$$|P(A)| = |P(B)| + |P(B)| - |P(B) \cap \{C \cup \{a\} \ldots\}|$$

$$= 2 \cdot |P(B)| - |\emptyset|$$

$$= 2 \cdot 2^{n} + 0 = 2^{n+1}$$

TABLE 1 Set Identities.

Identity	*Name*
$A \cup \emptyset = A$ $A \cap U = A$	Identity laws
$A \cup U = U$ $A \cap \emptyset = \emptyset$	Domination laws
$A \cup A = A$ $A \cap A = A$	Idempotent laws
$\overline{(\overline{A})} = A$	Complementation law
$A \cup B = B \cup A$ $A \cap B = B \cap A$	Commutative laws
$A \cup (B \cup C) = (A \cup B) \cup C$ $A \cap (B \cap C) = (A \cap B) \cap C$	Associative laws
$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$	Distributive laws
$\overline{A \cup B} = \overline{A} \cap \overline{B}$ $\overline{A \cap B} = \overline{A} \cup \overline{B}$	De Morgan's laws
$A \cup (A \cap B) = A$ $A \cap (A \cup B) = A$	Absorption laws
$A \cup \overline{A} = U$ $A \cap \overline{A} = \emptyset$	Complement laws

$$\overline{A \cup B} = \{x \mid x \notin (A \cup B)\}$$

$$= \{x \mid \neg(x \in A \cup B)\}$$

$$= \{x \mid \neg(x \in A \vee x \in B)\}$$

De Morgan

$$= \{x \mid \neg(x \in A) \wedge \neg(x \in B)\}$$

$$= \{x \mid x \notin A \wedge x \notin B\}$$

$$= \{x \mid x \in \overline{A} \wedge x \in \overline{B}\}$$

$$= \{x \mid x \in \overline{A} \cap \overline{B}\}$$

$$= \overline{A} \cap \overline{B}$$