Cartesian product

$A = \{1, 2\}$

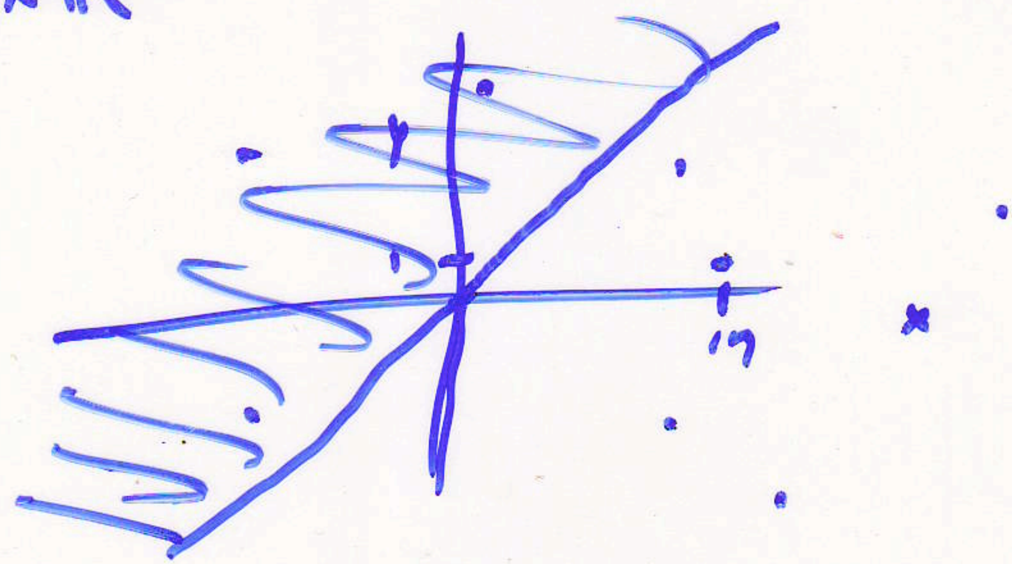
$B = \{a, b, c\}$

$A \times B$ = set of <u>ordered pairs</u>

$\{(1,a), (1,b), (1,c),$
$(2,a), (2,b), (2,c)\}$

$A \times B$ usually $\neq B \times A$

$\mathbb{R} \times \mathbb{R}$



$C \subseteq A \times B$ is a <u>relation</u>
between A & B

"$\leq$" = $\{(x,y) \mid$ Real x is less or = real $y\}$

12-1

$(A \times B) \times C \neq A \times (B \times C)$

$((1, a), \alpha) \qquad (1, (a, \alpha))$

$A \times B \times C =$ "$\times(A, B, C)$"

$(1, a, \alpha)$

Functions

$g: \mathbb{N} \to \mathbb{N}$ $f(x) = x/2$

$f: A \to B$

$f: \mathbb{N} \to \mathbb{N}$

$f(x) = x^2$

for every element of A
I associate 1 element of B

$A, B \neq \emptyset$

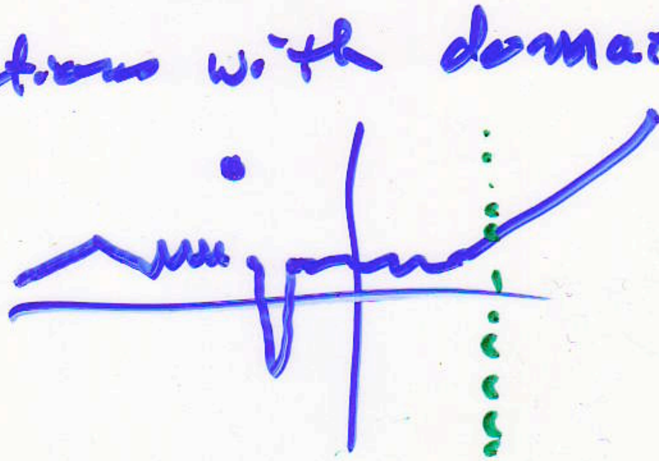
$\forall x \in A \; \underbrace{\exists!\, y \in B}$ st $y = f(x)$

$\exists y \in B$ st $y = f(x) \wedge$

$\exists y' \in B$ st $y' = f(x) \to y = y'$

~~fact~~ : A domain of f
B co-domain of f

for functions with domain & codomain $= \mathbb{R}$



"vertical line test"

$f(a) = b$

b is the <u>image</u> of a

a is ~~the~~ preimage of b

if $S \subseteq A$

$$f(S) = \{ y \mid f(x) = y \text{ for some } x \in S \}$$

$$= \{ f(s) \mid s \in S \}$$

$f : A \to B$

$f(A)$ is the <u>range</u> of f

$f(A) \subseteq B$

$g : N \to N$

$g(x) = 42 \; \forall x$

if range = codomain, then f is <u>onto</u>] Surjection

codomain of $g = N$

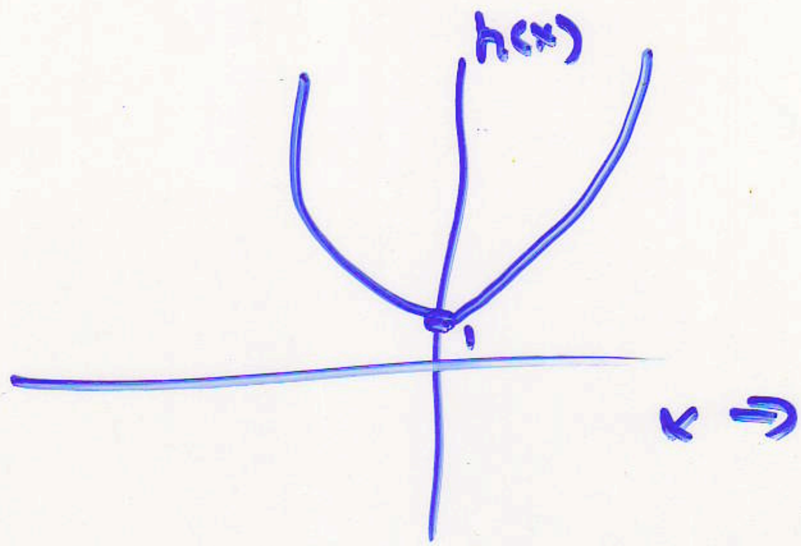
range of $g = \{42\}$

$g' : N \to \mathbb{R}$

$g'(x) = 42 \; \forall x$

12-4

$h \in R \to R \quad h(x) = 2x^2+1$



$\text{Range}(h) = \{ y \mid y \geq 1 \}$

$h' : \mathbb{R} \to \{ y \mid y \geq 1 \} \quad h(x) = 2x+1$

$k : N \to N \quad k(x) = 1/x$

$k(1) = 1$

$k(2) = 1/2$?

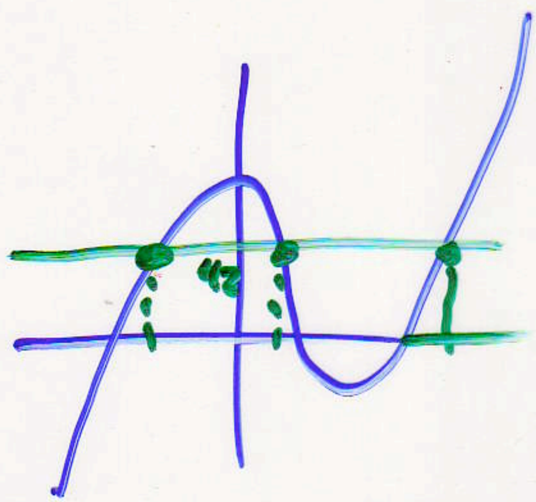
$k(0) = $?

Not A function

A function is ~~that~~
one to one
1-1
(an injection)

if $f(x) = f(y) \rightarrow x = y$

(contrapositive:
$x \neq y \rightarrow f(x) \neq f(y)$

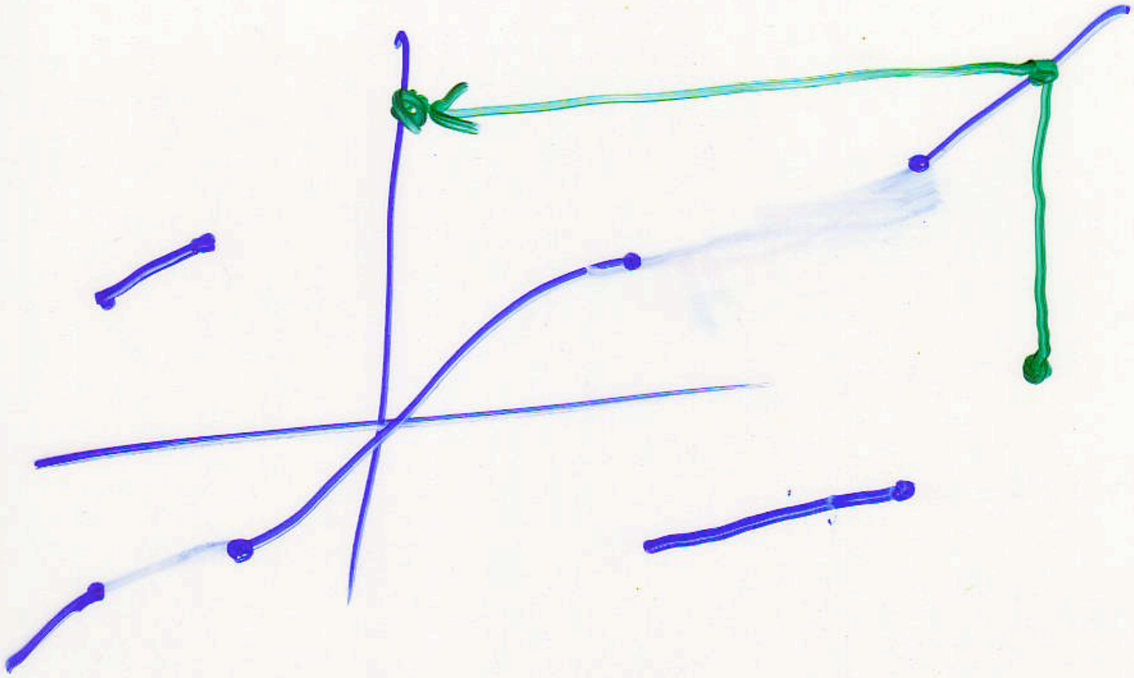


"horizontal line test"

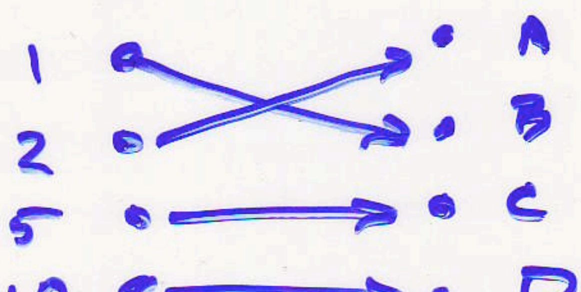
injective

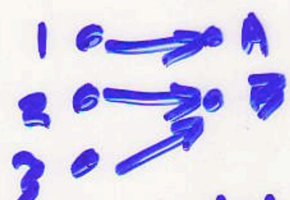
not

A <u>bijection</u> is both 1-1 & onto

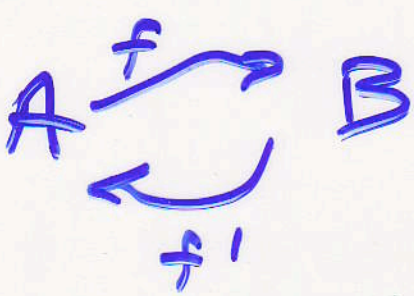


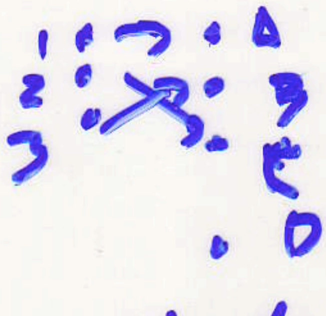
1-1 correspondence





not 1-1





not onto

if $f: A \to B$ is a bijection

Then can define function

$f^{-1}: B \to A$

defined by $f^{-1}(y) = x$ iff $f(x) = y$