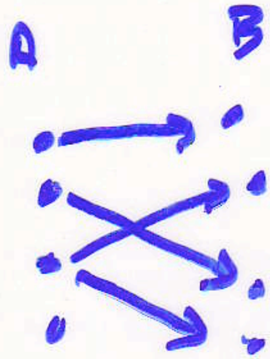
Finite A, B
$f: A \to B$



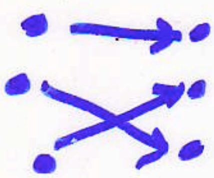
"1-many"
Not a function

onto

"into"

Many-1

1-1



bijection

Finite
if f is onto
$|A| \geq |B|$

2 sets A & B have equal cardinality if $\exists$ bijection $f: A \to B$

$A = \mathbb{N}$

$B =$ even natural #'s

$f(x) = 2x$ $\qquad |A| = |B|$

$f: A \to B$

if $b \in B$

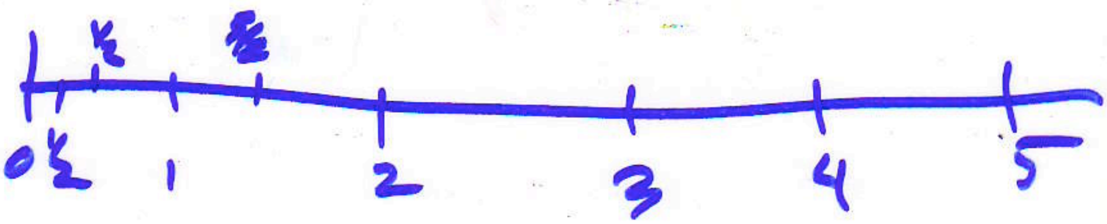
and $f(x) = b$ $\qquad 2x = b$

and $f(y) = b$ $\qquad 2y = b$

then $x = y$ $\qquad 2x = 2y$

$x = y$

A set is <u>countable</u> if its cardinality $= |A|$ for some $A \subseteq \mathbb{N}$

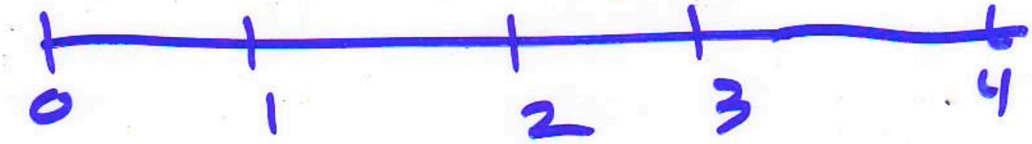


Fact $\mathbb{Q}^+$ (nonnegative rationals) are countable

	0	1	2	3	4	$\cdots$	
1	1st $0/1$	2nd $1/1$	$2/1$	4th $3/1$	$4/1$	$\cdots$	$24/1$
2	3rd $0/2$	5th $1/2$	$2/2$	$3/2$	$4/2$	$\cdots$	$24/2$
3	6th $0/3$	$1/3$	$2/3$	$\cdots$			
4	$0/4$	$1/4$	$2/4$	$\cdots$			
$\vdots$					$22/j$		

$f : \mathbb{N} \to \mathbb{Q}^+$

$f(i) =$ $i^{\underline{th}}$ distinct element of $\mathbb{Q}^+$ encountered in the order "diagonal" traversal above



Suppose $\mathbb{R}^+$ are countable

.... $[0, 1)$ ...

0	$0.\underline{0}000$
1	$0.2\underline{1}21212121\ldots$
2	$0.12\underline{3}4123456\ \ldots$
3	$0.314\underline{1}592653589793\ldots$
4	$0.2718\underline{2}818284\ \ldots$
5	$0.27190\underline{0}005000\ldots$
⋮	

$0.124281\ \ldots$

Let x differ from i^{th} row i^{th} digit.

Then $x \notin$ table.