Random Variable $\quad X : \text{SampleSpace} \to \mathbb{R}$

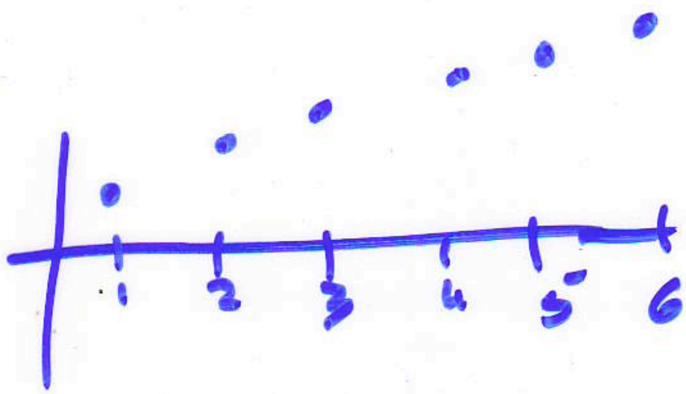
Distribution of X $\quad \{ (r, \text{Prob}(X=r)) \}$

$\quad \quad \sum_{s \text{ s.t. } X(s)=r} p(s)$

Eg. X = max of 2 dice

	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	2	3	4	5	6
3	3	3	3	4	5	6
4	4	4	4	4	5	6
5	5	5	5	5	5	6
6	6	6	6	6	6	6

$(1, 1/36)$
$(2, 3/36)$
$(3, 5/36)$
$(4, 7/36)$
$(5, 9/36)$
$(6, 11/36)$



Expected Value $\quad E(X) = \sum_{s \in S} X(s) \cdot p(s)$

≈ 4.47

$$= \sum_{r \in X(S)} r \cdot P(X=r)$$

Expected # of $\overset{p}{\text{successes}}$ in n Bernoulli Trials.

$$E(X) = \sum_{k=0}^{n} k \cdot P(X=k)$$

$$= \sum_{k=0}^{n} k \binom{n}{k} p^k q^{n-k} \qquad q = 1-p$$

$$= \sum_{k=1}^{n} k \binom{n}{k} p^k q^{n-k}$$

$$\sum_{k=1}^{n} \cancel{k} \cdot \frac{n(n-1)\cdots(n-k+1)}{\cancel{k} \cdots 2 \cdot 1} p^k q^{n-k}$$

$$= np \sum_{k=1}^{n} \frac{(n-1)\cdots(n-k+1)}{(k-1)\cdots 1} p^{k-1} q^{n-k}$$

$$= np \sum_{\bar{k}=0}^{n-1} \frac{n-1 \cdots (n-\bar{k})}{\bar{k} \cdots 1} \cdot p^{\bar{k}} q^{n-1-\bar{k}}$$

$$= np \sum_{\bar{k}=0}^{n-1} \binom{n-1}{\bar{k}} p^{\bar{k}} q^{(n-1)-\bar{k}}$$

$$= np\,(p+q)^{n-1} = np \cdot 1^{n-1} = np$$

Expectation is linear

→ $E(X+Y) = E(X) + E(Y)$

$E(aX+b) = a \cdot E(X) + b$

Pf

$$E(X+Y) = \sum_s p(s) \cdot (X(s) + Y(s))$$

$$= \sum_s \left(p(s) \cdot X(s) + p(s) \cdot Y(s)\right)$$

$$= \sum_s p(s) X(s) + \sum_s p(s) Y(s)$$

$$= E(X) + E(Y)$$

X_i = # of heads in i^{th} coin flip

$$E\left(\sum_i X_i\right) = \sum_i E(X_i) = \sum_i p = np$$

---

flip 1 coin, lay down n-1 others with same outcome.

$$E(X_1) = 1 \cdot p(X_1 = 1) + 0 \cdot p(X_1 = 0)$$
$$= p$$

$E(X_2) = p$

$E(\sum X_i) = np$

Variance

X_1 always 0

X_2 $-1, 0, +1$ w/ $p = 1/3$

X_3 $-10^6, +10^6$ $p = 1/2$

$E(X_1) = E(X_2) = E(X_3) = 0$

$$V(X) = E\left((X - E(X))^2 \right)$$

$V(X_1) = 0$

$V(X_2) = \frac{1}{3}(-1)^2 + \frac{1}{3}(0) + \frac{1}{3}(+1)^2 = \frac{2}{3}$

$V(X_3) = \frac{1}{2}(-10^6)^2 + \frac{1}{2}(+10^6)^2 = 10^{12}$

standard deviation $\sqrt{V(X)}$

$$V(x) = E(x^2) - (E(x))^2$$

$$E(x) = \mu$$

$$V(x) = E\left((x-\mu)^2\right)$$

$$= E(x^2 - 2x\mu + \mu^2)$$

$$= E(x^2) \underbrace{- 2\mu E(x)}_{-2\mu^2} + \mu^2$$

$$= E(x^2) - \mu^2$$

$$= E(x^2) - (E(x))^2$$