CSE 321: Discrete Structures
Assignment #3
January 22, 2009
Due: Wednesday, January 28, in Class

Reading Assignment: Sections 2.1 – 2.3, and 3.4 – 3.6.

Problems:

1. Section 1.6, exercise 28.
2. Prove that if you pick 10 numbers from 1 to 1000, then there is a pair of numbers such that the larger of the two is at most twice the other.
3. Let $Q(A, B)$ be the proposition that $A \subseteq B$. If the universe of discourse for both A and B is all sets of integers, what are the truth values of the following? Justify your answers.
   (a) $\forall A \exists B\ Q(A, B)$
   (b) $\forall B \exists A\ Q(A, B)$
   (c) $\exists A \forall B\ Q(A, B)$
   (d) $\exists B \forall A\ Q(A, B)$
4. Carefully prove the following implications (no Venn diagrams)
   (a) $(A \cup B = B) \rightarrow (A \subseteq B)$
   (b) $(A \subseteq B) \leftrightarrow (\bar{B} \subseteq \bar{A})$
5. Give an example of a function from $\mathbb{N}$ to $\mathbb{N}$ which is
   (a) one-to-one but not onto
   (b) onto but not one-to-one
   (c) both onto and one-to-one, but not the identity function
   (d) neither one-to-one nor onto

More on next page

The next two problems use the following denition: Let g be a function from the set A to the set B and let f be a function from the set B to the set C. The *composition* of the functions f and g, denoted by $f \circ g$, is dened by

$$(f \circ g)(a) = f(g(a)) \tag{1}$$

6. Let $f : \Re \to \Re$, where $f(x) = x^3$ and $g : \Re \to \Re$, where $g(x) = x - 1$. Give expressions for $f \circ f$, $f \circ g$, $g \circ f$, $g \circ g$.

7. **Extra credit:** If f and $f \circ g$ are one-to-one, does it follow that g is one-to-one? Justify your answer.