CSE 321: Discrete Structures
Assignment #5
February 15, 2009
Due: Friday, February 20, in class

Reading Assignment: Sections 5.1 – 5.4.

Justify your answers to the following problems.

Problems:

1. Assume that friendship is always mutual; that is, if A is a friend of B then B is also a friend of A. Show that under this assumption in any group of people there are always two people who have exactly the same number of friends within the group.

2. An ice cream parlor has 28 different flavors, 8 different kinds of sauce, and 12 toppings.

   (a) In how many different ways can a dish of three scoops of ice cream be made where each flavor can be used more than once and the order of the scoops does not matter?

   (b) How many dierent kinds of small sundaes are there if a small sundae contains one scoop of ice cream, a sauce, and a topping?

   (c) How many dierent kinds of large sundaes are there if a large sundae contains three scoops of ice cream, where each flaor can be used more than once and the order of the scoops does not matter; two kinds of sauce, where each sauce can be used only once and the order of the sauces does not matter; and three toppings, where each topping can be used only once and the order ot toppings does not matter?

3. Solve the following counting problems. In each case show the reasoning that led you to the answer.

   (a) A palindrome is a word that reads the same forwards and backwards. How many seven-letter palindromes can be made from the English alphabet?

   (b) How many bit strings of length 10 begin with three 0s or end with two 1s?

   (c) How many bit strings of length 10 contain either five consecutive 0s or five consecutive 1s?

   (d) Suppose you have n beads, each of a different color, that you need to string into a necklace? How many distinct necklaces can you make? (A necklace flipped over remains the same and does not count as a distinct necklace.)

   (e) How many ways can three distinct numbers be chosen from $1, 2, \ldots, 100$ such that their sum is even?

   More on next page.

4. **Extra credit**: Imagine a town with East-West streets numbered 1 through n, and North-South avenues numbered 1 through m. A taxi cab picks up a passenger at the corner of 1st street and 1st avenue. The passenger wishes to be delivered to n-th street and m-th avenue. It is quite clear that the passenger will be angry if the cab chooses a route longer than (n-1)+(m-1) blocks, so we wont allow the cabby to take a route longer than this. In other words, the cabby must always be increasing his street number or his avenue number. Suppose that there is an accident at i-th street and j-th avenue. How many routes can the cabby take that avoid the intersection with the accident?