Discrete Structures

Sets

Chapter 2, Sections 2.1–2.2

Dieter Fox

Sets

- $a \epsilon A$**:** Objects in a set are called elements / members of the set.

- **Set descriptions:** List all elements, set builder notation, Venn diagram

- $A = B$**:** Two sets A and B are equal if and only if they the same elements.

- $A \subseteq B$**:** The set A is subset of B if and only if every element of A is also an element of B.

- $A \subset B$**:** The set A is called proper subset of B if $A \subseteq B$ and $A \neq B$.

- $|S|$**:** If there are exactly n distinct elements in S where n is a nonnegative integer, we say that S is a finite set and that n is the cardinality of S. A set is said to be infinite if it is not finite.

Sets

- $P(S)$**:** The power set of S is the set of all subsets of the set S.

- The ordered n-tuple $(a_1, a_2, \ldots, a_n)$ is the ordered collection that has a_1 as its first element, a_2 as its second element, ..., and a_n as its ntth element.

- $A \times B$**:** The Cartesian product of A and B is the set of all ordered pairs (a, b) where $a \epsilon A$ and $b \epsilon B$.

- $A_1 \times A_2 \times \ldots \times A_n$**:** The Cartesian product of the sets $A_1, A_2, \ldots, A_n$ is the set of ordered $n - tuples(a_1, a_2, \ldots, a_n)$, where a_i belongs to A_i for $i = 1, 2, \ldots, n$.

Set operations

- $A \cup B$: The union of A and B is the set that contains all elements that are in A or in B.
- $A \cap B$: The intersection of A and B is the set that contains all elements that are in both A and B.
- Two sets are called disjoint if their intersection is the empty set ($\emptyset$).
- $A - B$: The difference of A and B is the set containing those elements that are in A but not in B. The difference of A and B is also called the complement of B wrt. A.
- $\bar{A}$: Let U be the universal set. The complement of A is the complement of A wrt. U.
- The union (intersection) of of a collection of sets is the set that contains those elements that are member of at least one (all) set(s) in the collection.

Set identities

$A \cap U = A$ $A \cup \emptyset = A$	Identity laws
$A \cup U = U$ $A \cap \emptyset = \emptyset$	Domination laws
$A \cup A = A$ $A \cap A = A$	Idempotent laws
$\overline{(\overline{A})} = A$	Double negation law
$A \cup B = B \cup A$ $A \cap B = B \cap A$	Commutative laws
$(A \cup B) \cup C = A \cup (B \cup C)$ $(A \cap B) \cap C = A \cap (B \cap C)$	Associative laws
$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$	Distributive laws
$\overline{(A \cap B)} = \overline{A} \cup \overline{B}$ $\overline{(A \cup B)} = \overline{A} \cap \overline{B}$	De Morgan's laws
$A \cup (A \cap B) = A$ $A \cap (A \cup B) = A$	Absorption laws
$A \cup \overline{A} = U$ $A \cap \overline{A} = \emptyset$	Complement laws