CSE 322 Lecture 3: Review of Proof Techniques

- Last Time:
  - **Proof by counterexample**: Give an example that disproves the given statement
  - **Proof by contradiction**: Assume statement is false and show that it leads to a contradiction
  - **Proof of set equality** $A = B$: Show $A \subseteq B$ and $B \subseteq A$
- Today (and beyond):
  - Proof of "X iff Y" (or $X \Leftrightarrow Y$) statements
  - Proof by construction
  - Proof by induction
  - "Bird-based" techniques: Pigeonhole principle and Dovetailing
  - CS Theoretician's favorite: Diagonalization

Proof Techniques II: The Big picture

- **Proving "X iff Y" statements**: Prove $X \Rightarrow Y$ ("X only if Y") and $Y \Rightarrow X$ ("X if Y")
  - Example: For all real numbers x, show $\lfloor x \rfloor = \lceil x \rceil$ iff $x \in Z$
- **Proof by construction**: Show that a statement can be satisfied by constructing an object using what is given
  - Example: Show that for all c, $\exists n_0$ s.t. $n^2 > cn$ for all $n \geq n_0$
- **Proof by induction** (very common in CS Theory): 2 steps –
  1. Basis Step: Show statement is true for some finite value n_0, typically $n_0 = 0$
  2. Induction hypothesis and induction step: Assume statement is true for some fixed but arbitrary $n \geq n_0$. Show it is also true for $n + 1$
  - Example: Show that for all $n \geq 0$, $1 + 2 + \ldots + n = n(n+1)/2$

The "Avian" Techniques

- **Pigeonhole principle**: If A and B are finite sets and $|A| > |B|$, then there is no one-to-one function from A to B
  - $f : A \rightarrow B$ is one-to-one if for any distinct $x, y \in A$, $f(x) \neq f(y)$
  - Idea: "more pigeons than pigeonholes" → at least one pigeonhole contains two pigeons. Prove by induction on $|B|$
  - E.g. In a room of 13 or more people, at least 2 have same birthmonth
- **Dovetailing**: Useful for showing union of any finite or countably infinite collection of countably infinite sets is again countably infinite
  - A is *countably infinite* if there is a 1-1 correspondence ("bijection") between N (the set of natural numbers) and A
  - E.g. Use dovetailing to show Z and $N \times N$ are both countably infinite

Next Class: Enter the finite automaton…

- Next time:
  - Infinite sets that are not countably infinite (diagonalization)
  - Finite automata 101
- Things to do over the weekend:
  - Browse course website
  - Sign up for mailing list (instructions on website)
  - Finish Chapter 0 and start Chapter 1
  - Start (and finish?) homework #1
  - Have a great weekend!