CSE 322: Regular Expressions and Finite Automata

- **Last Time**: Definition of a Regular Expression
  - R is a regular expression iff
    R is a string over $\Sigma \cup \{ \varepsilon, \emptyset, (,), \cup, * \}$ and R is:
    1. Some symbol $a \in \Sigma$, or
    2. ε, or
    3. $\emptyset$, or
    4. $(R1 \cup R2)$ where R1 and R2 are regular exps., or
    5. $R1R2 = R1 \circ R2$ where R1 and R2 are reg. exps., or
    6. $R1^*$ where R1 is a regular expression.
- **Precedence**: Evaluate * first, then $\circ$, then $\cup$
  - E.g. $0 \cup 11^* = 0 \cup (1 \circ (1^*)) = \{0\} \cup \{1, 11, 111, \ldots\}$

Examples

- What is R for each of the following languages?
  1. L(R) = {w | w contains exactly two 0's}
  2. L(R) = {w | w contains at least two 0's}
  3. L(R) = {w | w contains an even number of 0's}
  4. L(R) = {w | w does not contain 00}
  5. L(R) = {w | w is a valid identifier in C}

Regular Expressions and Finite Automata

- What is the relationship between regular expressions and DFAs/NFAs?
- Specifically:
  1. Given a reg. exp. R, can we create an NFA N such that L(R) = L(N)?
  2. Given an NFA N (or its equivalent DFA M), can we come up with a reg. exp. R such that L(M) = L(R)?

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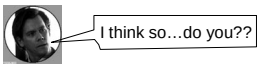


From Regular Expressions to NFAs

- Problem: Given *any* regular expression R, how do we construct an NFA N such that L(N) = L(R)?
- Soln.: Use the multi-part definition of regular expressions!!
  - Show how to construct an NFA for each possible case in the definition: R = *a*, or R = ε, or R = ∅, or R = (R1 ∪ R2), or R = R1°R2, or R = R1*.

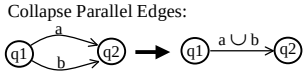
- Example: Draw NFA for 01Σ*10

From NFAs/DFAs to Regular Expressions

- Problem: Given *any* NFA N, how do we construct a regular expression R such that L(N) = L(R)?
- Solution: First, convert NFA N to an equivalent DFA M to keep things simple. Then:
  - Idea: Collapse 2 or more edges in M labeled with single symbols to a *new edge* labeled with an *equivalent regular expression*
  - This results in a "generalized" NFA (GNFA)
  - Our goal: Get a GNFA with 2 states (start and accept) connected by a single edge labeled with the required regular expression R

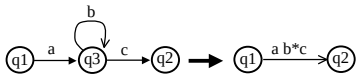
From NFAs/DFAs to Regular Expressions

- Steps for extracting regular expressions from DFAs:
  1. Add new start state connected to old one via an ε–transition
  2. Add new accept state receiving ε–transitions from all old ones
  3. Keep applying 2 rules until only start and accept states remain:
     1. Collapse Parallel Edges:



Note: Also applies to q1 = q2

     2. Remove "loopy" states:



Note: Also applies to q1 = q2

(Example: On board and in textbook)

Next Class: How to pump up them strings…

- Next time: Beyond the regular world…
  - Pumping Lemma for showing non-regularity of languages
- Things to do over the weekend:
  - Finish reading Chapter 1
  - Start (and finish?) homework #3
  - Watch a Kevin Bacon movie (optional)
  - Have a great weekend!