CSE 322
Intro to Formal Models in CS
Midterm Exam
Solution

Autumn 2000 — Handout 13

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1. Circle True or False below. *Very briefly justify your answers,* e.g. by giving a counter example, by citing a theorem we've proved, *briefly* sketching a construction, etc. Assume A and R are subsets of Σ^* for some fixed alphabet Σ.

   (a) If R is regular, and $A \subseteq R$, then A is regular. .... T F

   FALSE. Counterexample: $A = \{a^n b^n \mid n \geq 0\},\ R = \{a, b\}^*$.

   (b) If R is regular, and $R \subseteq A$, then A is regular. .... T F

   FALSE. Counterexample: $A = \{a^n b^n \mid n \geq 0\},\ R = \emptyset$.

   (c) If R is regular, and $A \cap R$ is regular, then A is regular. .... T F

   FALSE. Counterexample: $A = \{a^n b^n \mid n \geq 0\},\ R = \emptyset$.

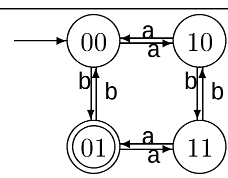
   (d) If R is regular, but $A \cap R$ is non-regular, then A is non-regular. .... T F

   TRUE, by closure of the class of regular languages under $\cap$.

   (e) If R is regular, then R^* is regular. .... T F

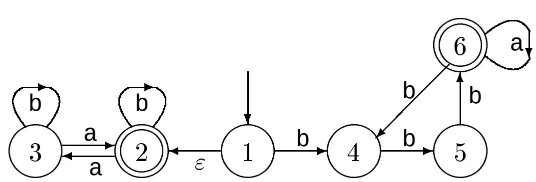
   TRUE, by closure of the class of regular languages under *.

2. Give a *deterministic* finite automaton recognizing the language $L = \{x \in \{a, b\}^* \mid x$ contains an even number of a's and an odd number of b's$\}$. E.g., b and $aaaba$ are in L, but $abab$ and $baaa$ are not. You do *not* need to give a correctness proof for your machine.



Note: State names indicate parity of #a, #b.

3. Consider the NFA $M = (Q, \Sigma, \delta, q_0, F)$ with the following transition diagram:



(a) In what states might the NFA be after reading input $bbba$? ______ 3, 6

(b) Does the NFA accept $bbba$? Why or why not? ______ Yes, since 6 is a final state.

(c) Suppose you apply the "subset" construction to build an equivalent DFA $M' = (Q', \Sigma, \delta', q_0', F')$. What state $q \in Q'$ would M' be in after reading the input $bbba$? ______ {3, 6}

(d) Is q above in F'? Why or why not? ______ Yes. It contains a final state of M.

(e) In terms of the states of M, what is the start state of M'? $q_0' =$ ______ $\{1, 2\}$

(f) What state is $\delta'(\{2, 4\}, a)$? ______ {3} ______ $\delta'(\{2, 6\}, a)$? ______ {3, 6} ______ $\delta'(\{5\}, a)$? ______ $\emptyset$

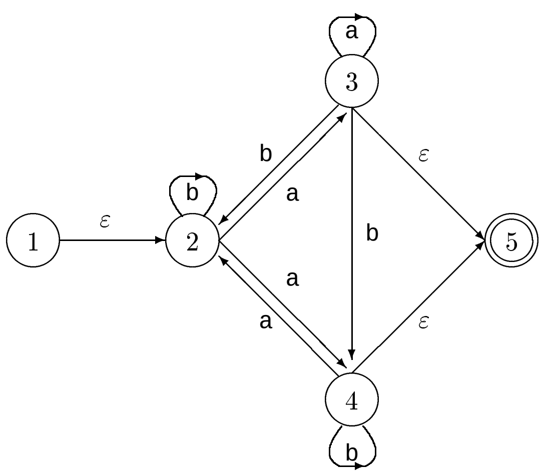
(g) Describe in English the language accepted by M. (Say *what* it is, not *how* M operates.)

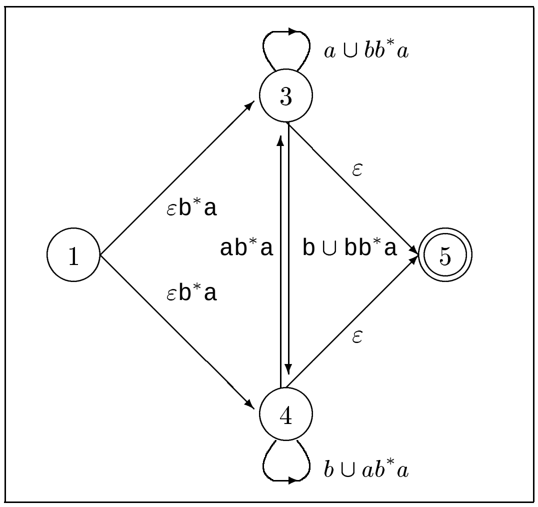
M accepts strings $x \in \{a, b\}^*$ with either

1) the number of b's in x, and the number of b's to the left of every a (if any) in x are positive multiples of 3, or
2) the number of a's in x is even.

FYI, a corresponding regular expression would be $(\texttt{bbba}^*)^* \cup \texttt{b}^*(\texttt{ab}^*\texttt{ab}^*)^*$.

4. Using the construction given in the text and lecture for converting an FA to a regular expression, eliminate state number 2 (and *only* state 2) from the following GNFA. The special start- and final-states have already been added. Arrows labeled $\emptyset$ are not shown. You may also omit them from your answer if you prefer, and you may simplify terms involving $\emptyset$ (e.g., $x \cup y \cdot \emptyset \equiv x$), but do *not* otherwise simplify the expressions.





5. Let $L = \{x \in \{a, b\}^* \mid x \text{ contains more } a\text{'s than } b\text{'s}\}$. Prove (using any method you wish) that L is not a regular language.

Assume L is regular. Let p be the pumping length for L, and let $s = a^p b^{p-1}$. Clearly s has more a's than b's, so it is in L, and $|s| \geq p$, so by the pumping lemma there must exist strings x, y, z such that $|xy| \leq p$, $|y| > 0$, and for all $i \geq 0$, $xy^iz \in L$. But $xy^0z = a^{p-|y|}b^{p-1} \notin L$, since $p - |y| \leq p - 1$. This contradicts the conclusion of the pumping lemma, and therefore L cannot be regular.