NAME: ______________________________

CSE 322
Intro to Formal Models in CS
Midterm Exam

Autumn 2000 — Handout 12

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SCORE	
1	/20
2	/20
3	/20
4	/20
5	/20
Total	/100

DIRECTIONS:

- Closed book, closed notes, except one page summary.
- Time limit 50 minutes.
- If possible, answer all problems on this sheet. (But attach separate sheets if necessary.)

1. Circle True or False below. *Very briefly justify your answers*, e.g. by giving a counter example, by citing a theorem we've proved, *briefly* sketching a construction, etc. Assume A and R are subsets of Σ^* for some fixed alphabet Σ.

   (a) If R is regular, and $A \subseteq R$, then A is regular. . . . . T F

   (b) If R is regular, and $R \subseteq A$, then A is regular. . . . . T F

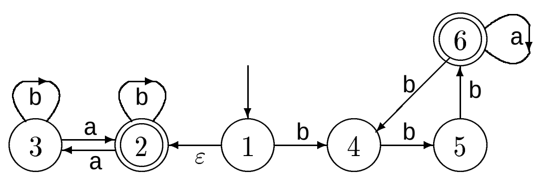
   (c) If R is regular, and $A \cap R$ is regular, then A is regular. . . . . T F

   (d) If R is regular, but $A \cap R$ is non-regular, then A is non-regular. . . . . T F

   (e) If R is regular, then R^* is regular. . . . . T F

2. Give a *deterministic* finite automaton recognizing the language $L = \{x \in \{a,b\}^* \mid x$ contains an even number of a's and an odd number of b's$\}$. E.g., b and $aaaba$ are in L, but $abab$ and $baaa$ are not. You do *not* need to give a correctness proof for your machine.

3. Consider the NFA $M = (Q, \Sigma, \delta, q_0, F)$ with the following transition diagram:



(a) In what states might the NFA be after reading input $bbba$? ________________

(b) Does the NFA accept $bbba$? Why or why not? ________________

(c) Suppose you apply the "subset" construction to build an equivalent DFA $M' = (Q', \Sigma, \delta', q'_0, F')$. What state $q \in Q'$ would M' be in after reading the input $bbba$? ______

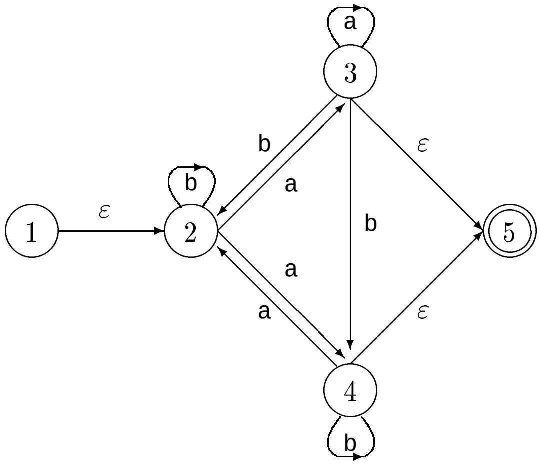
(d) Is q above in F'? Why or why not? ________________

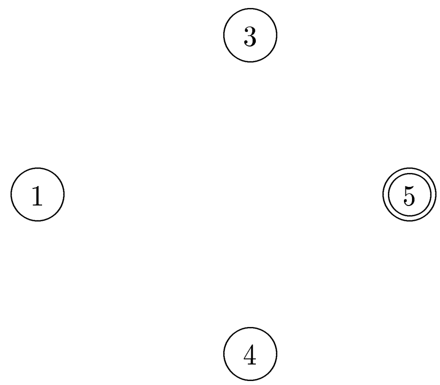
(e) In terms of the states of M, what is the start state of M'? $q'_0 =$ ________________

(f) What state is $\delta'(\{2,4\}, a)$? ______ ______ $\delta'(\{2,6\}, a)$? ______ ______ $\delta'(\{5\}, a)$? ______

(g) Describe in English the language accepted by M. (Say *what* it is, not *how* M operates.)

4. Using the construction given in the text and lecture for converting an FA to a regular expression, eliminate state number 2 (and *only* state 2) from the following GNFA. The special start- and final-states have already been added. Arrows labeled $\emptyset$ are not shown. You may also omit them from your answer if you prefer, and you may simplify terms involving $\emptyset$ (e.g., $x \cup y \cdot \emptyset \equiv x$), but do *not* otherwise simplify the expressions.





5. Let $L = \{x \in \{a, b\}^* \mid x \text{ contains more } a\text{'s than } b\text{'s } \}$. Prove (using any method you wish) that L is not a regular language.