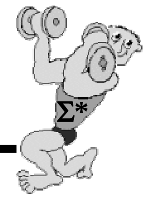
The Pumping Lemma for Regular Languages



- **What is it?**
  - A statement ("lemma") that is true for all regular languages
- **Why is it useful?**
  - Can be used to show that certain languages are *not regular*
  - How? By contradiction: Assume the given language is regular and show that it does not satisfy the pumping lemma
- **What is the idea behind it?**
  - Any regular language L has a DFA M that recognizes it
  - If M has p states and accepts a string of length ≥ p, the sequence of states M goes through must contain a cycle (repetition of a state) due to the pigeonhole principle! Thus:
  - *All strings* that make M go through this cycle 0 or any number of times are also accepted by M and should be in L.

Formal Statement of the Pumping Lemma

- **Pumping Lemma**: If L is a regular language, then there exists a number p (the "pumping length") such that for all strings s in L such that $|s| \geq$ p, there exist x, y, and z such that $s = xyz$ and:
  1. $xy^iz \in$ L for all $i \geq 0$, and
  2. $|y| \geq 1$, and
  3. $|xy| \leq$ p.
- **More Plainly**: p = number of states of a DFA accepting L. Any string s in L of length ≥ p can be expressed as $s = xyz$ where y is not null (y is the cycle), $|xy| \leq$ p (cycle occurs within p state transitions), and any "pumped up" string xy^iz is in L for all $i \geq 0$ (go through the cycle 0 or more times).
- Proved in 1961 by Bar-Hillel, Peries and Shamir.

The Pumping Lemma

- Proof on the board…(see page 79 in textbook)
  - See how it applies to {w | number of 0's in w is not divisible by 3}
- In-Class Examples: Using the pumping lemma to show a language L is *not regular*
  - 5 steps for a proof by contradiction:
  1. Assume L is regular.
  2. Let p be the pumping length given by the pumping lemma.
  3. Choose cleverly an *s* in L of length at least p, such that
  4. For *any way* of decomposing *s* into *xyz*, where $|xy| \le p$ and *y* isn't null,
  5. We can choose an $i \ge 0$ such that xy^iz is not in L.

Proving non-regularity as a Two-Person game

- An alternate view of using the pumping lemma to show a language L is not regular
  - Think of it as a *game between you and an opponent*:
  1. **You**: Assume L is regular
  2. **Opponent**: Chooses some value p
  3. **You**: Choose cleverly an *s* in L of length $\ge$ p
  4. **Opponent**: Breaks *s* down into some *xyz*, where $|xy| \le p$ and *y* is not null,
  5. **You**: Need to choose an $i \ge 0$ such that xy^iz is not in L (in order to win (the prize of non-regularity)!).
- See how this works for showing $\{0^n1^n \mid n \ge 0\}$ is not regular.

The Pumping Lemma Song (by Harry Mairson)

Any regular language *L* has a magic number *p*
And any long-enough word *s* in *L* has the following property:
Amongst its first *p* symbols is a segment you can find
Whose repetition or omission leaves *s* amongst its kind.

So if you find a language *L* which fails this acid test,
And some long word you pump becomes distinct from all the rest,
By contradiction you have shown that language *L* is not
A regular guy, resilient to the damage you have wrought.

But if, upon the other hand, *s* stays within its *L*,
Then either *L* is regular, or else you chose not well.
For *s* is *xyz*, and *y* cannot be null,
And *y* must come before *p* symbols have been read in full.

Source: http://www.cs.brandeis.edu/~mairson/poems/node1.html