CSE 322 **Winter 2000**

Midterm Examination

Name: ______________________________ Score: ____________

This examination is open book. You may consult your notes, returned homeworks, homework solutions, and the textbook. You may cite Theorems, Lemmas, and Examples from the book; you may not cite exercises from the book that were not assigned.

The entire examination is worth 80 points. The values of individual questions are marked. There are five questions.

1. [2×10 points] Prove or disprove the following for languages A and B:

   (a) [10 points] If A is regular and $A \cup B$ is regular, then B is regular.

   (b) [10 points] Same as part (a) above, except A and B are disjoint.

2. [10 points] For any language L over alphabet $\Sigma = \{0, 1\}$, consider the following language:

   - $\text{MIN}(L) = \{x \in L \mid \text{ no proper prefix of } x \text{ is in } L\ \}$

   Intuitively, $\text{MIN}(L)$ is the set of minimal strings in L, they have no prefix in L. For example, if $L = 0^*(11 + 1)$, then $\text{MIN}(L) = 0^*1$.

   Prove or disprove the following closure property of regular languages: for any regular language L, $\text{MIN}(L)$ is regular.

   If you prove your answer by construction, you need not prove that your construction is correct.

3. [10 points] For $a \in \Sigma$ and $w \in \Sigma^*$, $\#_a(w)$ is defined to be the number of occurrences of the symbol a in the string w. For example, $\#_a(abb) = 1$ and $\#_b(abb) = 2$. Consider the following language L:

$$L = \{w \in \{a, b, c\}^* \mid \#_a(w) + \#_c(w) \text{ is a prime}\}.$$

State and prove whether or not L regular. If you give an affirmative answer with a proof by construction, you need not prove that your construction is correct.

4. [10 points] Consider the context-free grammar $G = (V, \Sigma, P, S)$, where $V = \{S, A, B, C\}$, $\Sigma = \{a, b\}$, and P contains the following productions:

$$\begin{aligned} S &\to AB \mid CA \\ B &\to BC \mid AB \\ C &\to aB \mid b \\ A &\to aA \mid a. \end{aligned}$$

Give a description (in words or in notation) of $L(G)$.

5. [3 × 10 points] Let PAR be the language over the alphabet $\{(,)\}$ consisting of all strings of balanced parenthesis, i.e. each left parenthesis has a matching right parenthesis and pairs of matching parentheses are properly nested and the nesting depth could be arbitrary. For any integer $k \geq 1$, let PAR_k be the language that is a subset of PAR consisting only of strings in which the nesting depth is $\leq k$. For example, $(())() \in \text{PAR}_2$ and $((())) \notin \text{PAR}_2$ but $\in \text{PAR}_3$. Both strings are in PAR.

   (a) [10 points] Prove that PAR is context-free. Be sure to specify all four parts of your context-free grammer.

(b) [10 points] Prove or disprove: PAR is regular.

(c) [10 points] Prove or disprove: for any fixed k, PAR_k is regular.