CSE 322 **Winter 2000**

Final Examination

Name: ______________________________ Score: __________

This examination is open book. You may consult your class notes, homework assignments, handouts, homework solutions, and your textbook. You may cite Theorems, Lemmas, and Examples from the book; you may cite exercises from the book that were assigned as homework. You may **not** cite exercises from the book that were not on a homework assignment.

Keep your proofs succinct but complete. Write legibly. Use the back of pages if necessary.

There are 6 questions on 7 pages. The entire examination is worth 100 points. The values of individual questions and subquestions are marked.

1. [2×15 points] Consider the following language over the alphabet $\Sigma = \{a, b, c\}$.

$$L = \{a^i b^j c^k \mid i \neq j \text{ or } j \neq k \text{ or both}\}$$

   (a) [15 points] Prove that L is not regular.

   (b) [15 points] Prove that L is context-free.

2. [2×10 points] For any language $L \subseteq \{0,1\}^*$, consider the following definitions:

- EQUAL$(L) = \{x \in L \mid x$ has an equal number of 0's and 1's$\}$.
- DOUBLE$(L) = \{x \in L \mid x$ ends with 00 or 11$\}$.

Prove or disprove the following closure properties of regular languages. If you prove your answer by construction, you need not prove your construction is correct.

(a) [10 points] For any regular language L, EQUAL(L) is regular.

(b) [10 points] For any regular language L, DOUBLE(L) is regular.

3. [10 points] Recall that the class of regular languages is closed under Kleene star. Prove that the converse of this closure property is false (i.e. L^* regular does not imply L regular) by giving a counter example.

4. [15 points] Consider the context-free grammar $G = (V, \Sigma, P, S)$, where $V = \{S, A, B\}$, $\Sigma = \{a, b\}$, and P contains the following productions:

$$
\begin{aligned}
S &\rightarrow aB \mid bA \\
A &\rightarrow a \mid aS \mid bAA \\
B &\rightarrow b \mid bS \mid aBB.
\end{aligned}
$$

For the string $aaabbabbba$ show (i) a rightmost derivation, (ii) a leftmost derivation, and (iii) a parse-tree.

5. [15 points] State and prove whether or not the following language is context-free:

$$L = \{a^{2^i} \mid i \geq 2\}$$

If you give an affirmative answer with a proof by construction, you need not prove that your construction is correct.

6. [10 points] In class I presented an *encoding scheme* for Turing machines with $\Sigma = \{0, 1\}$. This is a method by which any such Turing machine can be encoded as a string in $\{0, 1\}^*$. The method is such that each string in $\{0, 1\}^*$ can be *decoded* as a unique Turing machine or as an invalid encoding. For the purposes of this problem, assume that we have a similar encoding for a *pair* of Turing machines. That is, any string $\langle M_1, M_2 \rangle$ in $\{0, 1\}^*$ that is in fact a valid encoding, can be uniquely decoded as a pair of Turing machines. The details of these encoding schemes are irrelevant to this problem and you should not concern yourself with them.

   Consider the following language $L = \{\langle M_1, M_2 \rangle \mid L(M_1) \cap L(M_2) \text{ is Turing-recognizable}\}$.

   State whether L is

   - decidable,
   - Turing-recognizable but not decidable, or
   - not Turing-recognizable.

   Argue briefly but convincingly for your answer.