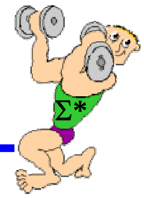
The Pumping Lemma for Regular Languages



- **What is the idea behind it?**
  - Any regular language L has a DFA M that recognizes it
  - If M has **p states** and accepts a string of length ≥ p, the sequence of states M goes through must contain a **cycle** (repetition of a state) due to the *pigeonhole principle*! Thus:
  - *All strings* that make M go through this cycle 0 or any number of times are also accepted by M and *should be in L.*

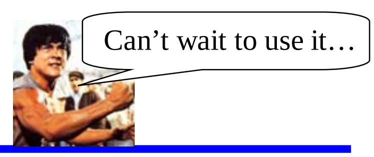
Formal Statement of the Pumping Lemma

- **Pumping Lemma**: If L is a regular language, then there exists a number p (the "pumping length") such that for all strings s in L such that $|s| \geq p$, there exist x, y, and z such that $s = xyz$ and:
  1. $xy^iz \in L$ for all $i \geq 0$, and
  2. $|y| \geq 1$, and
  3. $|xy| \leq p$.
- On board proof…(see page 79 in textbook)

Pumping Lemma in Plain English

- p = number of states of a DFA accepting L.
- Any string *s* in L of length ≥ p can be expressed as *s* = *xyz* where *y* is not null (*y* is the cycle), |*xy*| ≤ p (cycle occurs within p state transitions), and any “pumped” string xy^iz is in L for all *i* ≥ 0 (go through the cycle 0 or more times).

Using The Pumping Lemma



- In-Class Examples: Using the pumping lemma to show a language L is *not regular*
  - ➪ 5 steps for a proof by contradiction:
  1. Assume L is regular.
  2. Let p be the pumping length given by the pumping lemma.
  3. Choose cleverly an *s* in L of length at least p, such that
  4. For *any way* of decomposing *s* into *xyz*, where |*xy*| ≤ p and *y* isn't null,
  5. You can find an *i* ≥ 0 such that xy^iz is not in L.
- Example 1: $\{0^n1^n \mid n \geq 0\}$

Proving non-regularity as a Two-Person game

- An alternate view of using the pumping lemma to show a language L is not regular
  - Think of it as a *game between you and an opponent (KB)*:
  1. **You**: Assume L is regular
  2. **KB**: Chooses some value p
  3. **You**: Choose cleverly an *s* in L of length ≥ p
  4. **KB**: Breaks *s* down into some *xyz*, where $|xy| \leq p$ and *y* is not null,
  5. **You**: Need to choose an $i \geq 0$ such that xy^iz is not in L (in order to win (the prize of non-regularity)!).
- See how this works for showing $\{0^n1^m \mid n > m\}$ is not regular.
- Another example: Show ADD = {x=y+z | x, y, z are binary numbers and x is the sum of y and z} is not regular

Da Pumpin' Lemma

(Lyrics: Harry Mairson)

Hear it on my new album:
Dig dat funky DFA

Any regular language *L* has a magic number *p*
And any long-enough word *s* in *L* has the following property:
Amongst its first *p* symbols is a segment you can find
Whose repetition or omission leaves *s* amongst its kind.

So if you find a language *L* which fails this acid test,
And some long word you pump becomes distinct from all the rest,
By contradiction you have shown that language *L* is not
A regular guy, resilient to the damage you have wrought.

But if, upon the other hand, *s* stays within its *L*,
Then either *L* is regular, or else you chose not well.
For *s* is *xyz*, and *y* cannot be null,
And *y* must come before *p* symbols have been read in full.

 Source: http://www.cs.brandeis.edu/~mairson/poems/node1.html

If $\{0^n 1^n \mid n \geq 0\}$ is not Regular, what is it?

Enter…the world of Grammars