

You do not have to turn in this assignment, but it would be good practice to be sure you understand the recent material and for the exam.

1. Use the procedure of Lemma 2.15 to convert the pushdown automaton of Figure 2.8 into an equivalent grammar. Here are the steps you should follow:
  - (a)  $q_1$  needn't be in  $F$ , since this doesn't change the language accepted. (Why not?) This change leaves a unique accepting state, which we need for the conversion procedure.
  - (b) You will have to add a state  $p$  between states  $q_2$  and  $q_3$ . (Why?) What are the transitions from  $q_2$  to  $p$  and from  $p$  to  $q_3$ ? (You will save yourself a little effort later if you use a brand new stack symbol here.)
  - (c) Write down all the rules that the conversion procedure requires of the types  $A_{pq} \rightarrow aA_rsb$  and  $A_{pp} \rightarrow \varepsilon$ .
  - (d) The conversion procedure requires 125 rules of the third type (why?), and I won't make you write them all down. Instead, explain why you never need any rules of this third type if the pushdown automaton does all of its pushes before any of its pops. (Such a pushdown automaton is called a "one-turn pushdown automaton".)
  - (e) What is the set  $V$  of variables of your resulting grammar, and which is the start variable  $S$ ?
2. In Assignment 6 you converted the context-free grammar  $G_4$  given in Example 2.3 into Chomsky normal form. For this converted grammar, apply the dynamic programming procedure from the handout "Membership Testing in Context-Free Languages" to the input string  $(a + a) \times a$ . Fill in the  $7 \times 7$  table, and explain how you can tell at the end whether or not the input string is in the language.
3. Problem 2.18, parts (b) and (c).
4. Problem 2.20. (Problem 2.19 may help you get started with this.)
5. Problem 2.27. Note that the complement of  $D$  is *not* context-free, so this problem gives an explicit counterexample to closure under complement. (Why is the complement of  $D$  not the language of Example 2.22? Why is the complement not context-free?)