

CSE 322  
Autumn Quarter 2003  
Assignment 6  
Due Friday, November 14, 2003

All solutions should be neatly written or type set. All major steps in proofs and algorithms must be justified.

1. (10 points) In this problem you will explore how to convert an ambiguous context-free grammar into an unambiguous one. A context-free grammar is ambiguous if there is some  $w \in L(G)$  such that  $w$  has at least two distinct parse tree (or equivalently,  $w$  has at least two different leftmost derivations). Consider the context-free grammar  $G = (\{S\}, \{a, b\}, R, S)$  where

$$R = \{S \rightarrow SS, S \rightarrow aSb, S \rightarrow ab\}.$$

- (a) Demonstrate that  $G$  is ambiguous by showing two distinct parse trees for some string in  $L(G)$   
(b) Give an alternative grammar for  $L(G)$  that is not ambiguous.
2. (10 points) In this problem you will design a context-free grammar. Design a context-free grammars for the language

$$L = \{a^n b^m c^{2n+3m} : n, m \geq 0\}$$

and give a derivation of  $ab^2c^8$ . For each nonterminal in your grammar explain what set of strings is derived by the nonterminal. That is, for each nonterminal  $A$  describe the language  $\{w \in \{a, b, c\}^* : A \xRightarrow{*} w\}$ .

3. (10 points) In this problem you will explore some closure properties of context-free languages.
- (a) Show that the context-free languages are closed under union. That is, given two context free grammars  $G_1 = (V_1, \Sigma_1, R_1, S_1)$  and  $G_2 = (V_2, \Sigma_2, R_2, S_2)$  construct a new grammar  $G$  such that  $L(G) = L(G_1) \cup L(G_2)$ .
- (b) Show that the context-free languages are not closed under intersection. You can use the fact that the language  $\{0^n 1^n 2^n : n \geq 0\}$  is *not* context-free.
- (c) Show that the context-free languages are not closed under complement.