

PROBLEM SET 3  
Due Friday, April 25, 2003, in class

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1. Apply the state elimination procedure described in class to convert the finite automata (c) from Problem 2.3.7 on Page 84 of Lewis and Papadimitriou into a regular expression. Simplify the resulting regular expression as much as you can.
2. Write down regular expressions for each of the following languages over the alphabet  $\{0, 1\}$ .
  - (a)  $L_1 = \{w \mid w \text{ starts with } 0 \text{ and has odd length, or starts with } 1 \text{ and has even length}\}$ .
  - (b)  $L_2 = \{w \mid w \text{ is any string except } 00 \text{ or } 000\}$ .
  - (c)  $L_3 = \{w \mid w \text{ does not contain the substring } 110\}$ .
3. Lewis and Papadimitriou, Problem 1.8.4.  
(You **must** give a regular expressions based proof to receive credit, i.e., do **not** resort to a DFA/NFA construction to prove regularity of  $L'$ .)
4. Let  $r$  and  $s$  be regular expressions where the language represented by  $r$  does not contain the empty string  $\epsilon$ . Consider the equation  $X = r \circ X \cup s$  (where  $\circ$  stands for concatenation of regular expressions, and  $\cup$  for union) with unknown variable  $X$ . Find a solution (namely, a regular expression) for  $X$  that satisfies the above equation and prove that this solution is unique. (Comment: *This question is harder than the others!*)
5.
  - (a) By first constructing an NFA and then adding the necessary backward transitions, construct the DFA useful for determining whether the pattern  $aababaabaaa$  occurs in a string over the alphabet  $\{a, b\}$ .
  - (b) Lewis and Papadimitriou, Problem 2.6.3, Part (c).
  - (c) (Just for fun; no need to turn anything in for this part) Think about why such an NFA as in (b) above is good to have; this is actually Part (d) of Problem 2.6.3. (Again, you **don't have to** turn in a solution to this part.)