CSE 322
Winter Quarter 2003
Assignment 6
Due Friday, February 21, 2003

All solutions should be neatly written or type set. All major steps in proofs and algorithms must be justified.

1. (10 points) In this problem you will explore how to convert an ambiguous context-free grammar into an unabibuous one. A context-free grammar is ambiguous if there is some $w \in L(G)$ such that w has at least two distinct parse tree (or equivalently, w has at least two different leftmost derivations). Consider the context-free grammar $G = (\{S\}, \{a, b\}, R, S)$ where

$$R = \{S \to SS, S \to aSb, S \to ab\}.$$

   (a) Demonstrate that G is ambiguous by showing two distinct parse trees for some string in $L(G)$

   (b) Give an alternative grammar for $L(G)$ that is not ambiguous.

2. (10 points) In this problem you will design a context-free grammar. Design a context-free grammars for the language

$$L = \{a^n b^m c^{2n+m} : n, m \geq 0\}$$

   and give a derivation of $abbcccc$. For each nonterminal in your grammar explain what set of strings is derived by the nonterminal. That is, for each nonterminal A describe the language $\{w \in \{a, b, c\}^* : A \stackrel{*}{\Rightarrow} w\}$.

3. (10 points) A context-free grammar $G = (V, \Sigma, R, S)$ is *right-linear* if every production in R has one of the following forms (i) $A \to wB$ where $A, B \in V$ and $w \in \Sigma^*$ or (ii) $A \to w$ where $A \in V$ and $w \in \Sigma^*$.

   (a) Consider the right-linear grammar $G = (V, \Sigma, R, S)$ where

$$\begin{aligned} V &= \{S, A, B\} \\ \Sigma &= \{0, 1\} \\ R &= \{S \to 00S, S \to 11A, S \to \varepsilon, A \to 1A, A \to 100B, \\ &\quad B \to 0B, B \to 11S\} \end{aligned}$$

   Design an NFA that accepts $L(G)$.

   (b) Prove generally that the language generated by a right-linear grammar is always regular. Start with a right-regular grammar $G = (V, \Sigma, P, S)$. Then define a NFA that accepts the same language that the grammar generates.