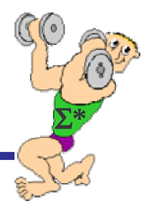
Beyond the Regular world…

- Are there languages that are *not* regular?
- **Idea:** If a language violates a property obeyed by all regular languages, it cannot be regular!
  - **Pumping Lemma** for showing *non-regularity* of languages

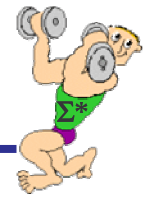
http://www.ipjnet.com/schwarzenegger2/pages/arnold_01.htm

The Pumping Lemma for Regular Languages



- **What is it?**
  - A statement (“lemma”) that is true for all regular languages
- **Why is it useful?**
  - Can be used to show that certain languages are *not regular*
  - How? *By contradiction*: Assume the given language is regular and show that it does not satisfy the pumping lemma

More about the Pumping Lemma



- **What is the idea behind it?**
  - Any regular language L has a DFA M that recognizes it
  - If M has **p states** and accepts a string of length ≥ p, the sequence of states M goes through must contain a **cycle** (repetition of a state) due to the *pigeonhole principle*! Thus:
  - *All strings* that make M go through this cycle 0 or any number of times are also accepted by M and *should be in L*.

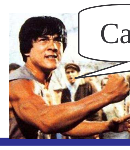
Formal Statement of the Pumping Lemma

- **Pumping Lemma**: If L is regular, then ∃ p such that ∀ *s* in L with |*s*| ≥ p, ∃ *x*, *y*, *z* with *s* = *xyz* and:
  1. $xy^iz \in L \; \forall i \geq 0$, and
  2. $|y| \geq 1$, and
  3. $|xy| \leq p$.
- Proof on board…(see also page 79 in textbook)
- Proved in 1961 by Bar-Hillel, Peries and Shamir

Pumping Lemma in Plain English

- Let L be a regular language and let p = "pumping length" = no. of states of a DFA accepting L
- Then, any string *s* in L of length ≥ p can be expressed as *s* = *xyz* where:
  - *y* is not empty (*y* is the cycle)
  - |*xy*| ≤ p (cycle occurs within p state transitions), and
  - any "pumped" string xy^iz is also in L for all *i* ≥ 0 (go through the cycle 0 or more times)

Using The Pumping Lemma



- In-Class Examples: Using the pumping lemma to show a language L is *not regular*
  - 5 steps for a proof by contradiction:

1. Assume L is regular.
2. Let p be the pumping length given by the pumping lemma.
3. Choose cleverly an *s* in L of length at least p, such that
4. For *all ways* of decomposing *s* into *xyz*, where |*xy*| ≤ p and *y* is not null,
5. There is an *i* ≥ 0 such that xy^iz is not in L.

Proving Non-Regularity using the Pumping Lemma

- Examples: Show the following are not regular
  - $L_1 = \{0^n1^n \mid n \geq 0\}$ over the alphabet {0, 1}
  - L_2 = {w | w contains equal number of 0s and 1s} over the alphabet {0, 1}
- Try these at home:
  - $L_3 = \{0^n1^m \mid n > m\}$ over the alphabet {0, 1}
  - ADD = {x=y+z | x, y, z are binary numbers and x is the sum of y and z} over the alphabet {0, 1, =, +}
  - PRIMES = $\{0^p \mid p$ is prime$\}$ over the alphabet {0}