CSE 322 Autumn 2004

Homework Assignment # 1

Due Date: Wednesday, October 13 (at the *beginning* of class)

1. (25 points) Find an expression for each of the following sets involving only A, B, C, and the operations $\cup$, $\cap$, and complement $\overline{\ }$. You may use parentheses (…) as needed.
   Example: {x | x is in A but not in B} = $A \cap \overline{B}$

   a. {x | x is in A or x is in both B and C}
   b. {x | x is in B or x is in C but not in all three sets A, B, and C}
   c. {x | x is in exactly one of the three sets A, B, and C}
   d. {x | x is in at most one of the three sets A, B, and C}
   e. {x | x is in exactly two of the three sets A, B, and C}

2. (20 points) Let A be the set of natural numbers between 10 and 20 <u>divisible by</u> 3. Let B be the set of <u>odd</u> natural numbers between 10 and 18.
   a. Which of the following statements is/are true: $A \subseteq B$, $B \subseteq A$, $A \cap B \neq \emptyset$, $A \subseteq \{n \mid n$ is divisible by 5 or n is divisible by 6$\}$
   b. Prove or disprove: $A \times B = B \times A$
   c. What is A^3? (List all the elements)
   d. What is the power set of B-A? (Recall: $B-A = B \cap \overline{A}$)

3. (15 points) Prove, using the definition of set equality, that for all sets A and B: $(A \cap B) \cup (A-B) = A$.

4. (20 points) Prove or disprove: For any natural number `n`, the union of `n` countably infinite sets is also countably infinite. (Hint: Read pages 161-162 in the text and the handout to be given in class on Friday).

5. (20 points) Your boss at Googyall.com isn't convinced the code fragment you wrote is correct. You decide to put your 322 knowledge to good use (since your job is on the line). Prove, using induction, that for any `x`, the following code fragment you wrote prints out $(x+n)^2$ for any natural number `n` (`sqrt` computes the square root of a number):

```
y = x*x;
for i = 1 to n
   y = y + 2*sqrt(y) + 1;
print(y);
```