 CSE 322: The Last Homework Assignment

Due Date: Wednesday, December 8 (at the *beginning* of class)
Note: Finals on Monday, December 13, 2:30-4:20 pm in class

1. (30 points) Closure properties:
    a. Show that decidable languages are closed under:
        i. complementation
        ii. concatenation
    b. Show that Turing-recognizable languages are closed under the <u>union</u> operator.
    Give *implementation level details* of the necessary Turing machines in each case.

2. (40 points) Exercises <u>4.3</u> and <u>4.4</u> in the textbook. (Hint: Construct your decider TMs based on deciders constructed in proofs for theorems in Section 4.1. Explain your proof idea and then give your TM in the format M = "On input …: ").

3. (30 points) Let $ALL_{TM} = \{\langle M \rangle \mid M$ is a TM and $L(M) = \Sigma^*\}$. Show that ALL_{TM} is undecidable by giving a reduction from a known undecidable language to ALL_{TM}. For your reduction, you may use A_{TM} or any of the languages shown to be undecidable in Section 5.1 in the textbook (pages 171-176 only).

Bonus question (no points!): Is the question "Does God exist?" decidable? (Hint: See Problem 3.19 in the textbook)