CSE 322: Introduction to Formal Models in Computer Science
Assignment #7
November 28, 2005
due: Monday, December 5

1. Use the procedure of Lemma 2.27 [1st Ed: Lemma 2.15] to convert the pushdown automaton of Figure 2.19 [1st Ed: Figure 2.8] into an equivalent grammar. Here are the steps you should follow:

   (a) q_1 needn't be in F, since this doesn't change the language accepted. Explain why not. This change leaves a unique accepting state, which we need for the conversion procedure.

   (b) You will have to add a state p between states q_2 and q_3. Explain why. What are the transitions from q_2 to p and from p to q_3? (You will save yourself a little effort later if you use a brand new stack symbol here.)

   (c) Write down all the rules that the conversion procedure requires of the types $A_{pq} \to aA_{rs}b$ and $A_{pp} \to \varepsilon$.

   (d) The conversion procedure requires 125 rules of the third type (why?), and I won't make you write them all down. Instead, explain why you never need any rules of this third type if the pushdown automaton does all of its pushes before any of its pops. (Such a pushdown automaton is called a "one-turn pushdown automaton".)

   (e) What is the set V of variables of your resulting grammar, and which is the start variable S?

2. In Assignment 5 you converted the context-free grammar G_4 given in Example 2.4 [1st Ed: Example 2.3] into Chomsky normal form. For this converted grammar, apply the dynamic programming procedure from the handout "Membership Testing in Context-Free Languages" to the input string $(a + a) \times a$. Fill in the 7×7 table, and explain how you can tell at the end whether or not the input string is in the language.

3. Let B be the language of all palindromes over $\{0, 1\}$ containing an equal number of 0s and 1s. Show that B is not context-free.

4. Let $P = \{a^n \mid n \text{ is a prime number}\}$ over the alphabet $\Sigma = \{a\}$. Prove that P is not context-free. [Hint: see the solution to the midterm exam, problem 5(b).]