CSE 322: Formal Models in Computer Science Winter 2005

PROBLEM SET 2
Due Friday, January 21, 2005, in class

Reading assignment: Sipser's book, sections 1.2 and 1.3.

Instructions: Same as for Problem set 1.

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1. Exercise 1.10, Sipser's textbook. (Swapping accept and non-accept states in DFAs and NFAs)

2. Recall the subset construction converting NFAs to DFAs that we discussed in class. If N was the original NFA with start state s, and Δ, s' were the transition function and start state respectively of the constructed DFA D, give a formal proof by induction of the following claim (which we used to argue that the construction worked):

$$\text{For all } x \in \Sigma^*,\ \widehat{\Delta}(s', x) = \mathsf{Reach}_N(s, x).$$

3. Use the subset construction to convert the NFA in Exercise 1.12 (b) of Sipser's book to an equivalent DFA.

4. (20 points) Construct an **NFA** (giving your answer in form of a state diagram) as well as a **regular expression** for each of the following languages over the alphabet $\{0, 1\}$:

   (a) $L_1 = \{w \mid w \text{ is any string except 11 and 111}\}$.

   (b) $L_2 = \{w \mid w$ contains an equal number of occurrences of the substrings 01 and 10$\}$. (Thus $101 \in L_2$ because 101 contains a single 01 and a single 10, but $1010 \notin L_2$ because 1010 contains two 10s and one 01.)

   (c) $L_3 = \{w \mid w$ contains two 0's that are separated by a number of positions that is a multiple of 3$\}$. (Note that 0 is also a multiple of 3.)

5. Suppose that L is regular, i.e. is recognized by a DFA. Then:

   (a) Prove that $L^R = \{w^R \mid w \in L\}$ is also regular. (Here w^R denotes the *reverse* of the string w, i.e. if $w = w_1 w_2, \ldots, w_n$ then $w^R = w_n w_{n-1} \ldots w_2 w_1$.)

   (b) Prove that $\mathsf{MIN}(L) = \{w \in L \mid \text{no proper prefix of } w \text{ is in } L\}$ is also regular.

   (c) * (Extra Credit) Define $\mathsf{HALF}(L) = \{x \mid \text{for some } y \text{ such that } |x| = |y|,\ xy \text{ belongs to } L\}$. Prove that $\mathsf{HALF}(L)$ is regular if L is regular.
   (For all of the parts above, presenting a *correct* construction of an NFA/DFA for the languages in question, along with an informal yet convincing explanation of why it does the right thing, will suffice. Fully formal proofs by induction are *not* required.)