

PROBLEM SET 4
Due Friday, April 28, 2006, in class

Reading Assignment: Handouts on Myhill-Nerode theorem and DFA minimization.

Instructions: The basic instructions are the same as in Problem Set 1.

There are **FOUR** questions in this assignment. Again, no harm in starting early!

Do not forget to mention the names of your collaborators in your homework.

1. ($3 \times 10 = 30$ points) Prove the following languages are not regular using the pumping lemma.

(a) (*) $L_1 = \{www \mid w \in \{0,1\}^*\}$.

(b) $L_2 = \{a^n \mid n \text{ is a prime}\}$.

(c) $L_3 = \{w \mid w \in \{0,1\}^* \text{ such that } w = x1y, \text{ where } x \text{ is the binary representation of a non-negative integer } n \text{ and } y \text{ is a sequence of } n \text{ 0s}\}$.

For example, $110, 101100000 \in L_3$ while $1011, 10100000 \notin L_3$.

2. (*) ($10 + 8 + 2 = 20$ points) Show that the language

$$\{a^i b^j c^k \mid i, j, k \geq 0, \text{ and if } i = 1 \text{ then } j = k\}$$

satisfies the conclusion of the pumping lemma.

Prove that the above language is not regular. Thus, this is an example where the pumping lemma cannot prove that a language is not regular.

Does this example contradict the pumping lemma ? Briefly justify your answer.

3. ($2 \times 10 = 20$ points) Use the method from the Myhill-Nerode handout to prove that the languages L_1 and L_3 (in problem 1) are not regular.

4. (10 points) (**Bonus**) Sipser, Problem 1.56, Page 91 (Problem 1.40, Pg. 90 in first edition).