Myhill-Nerode theorem

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Announcements

- Handout
  - Solutions to H/W# 3
- Pick up handout on DFA minimization
  - If you did not do so last class

Errata for Soln to H/W #3

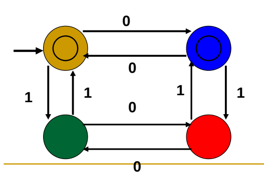
- Solution to 3(b) is the solution for 3(a)
- Solution for 3(b) is the following
- $1(\Sigma\ \Sigma)^*\ \cup\ 010(\Sigma\ \Sigma\ \Sigma)^*$

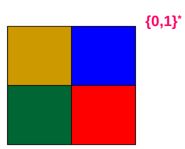
Puzzle

- Prove that any DFA for the following language has at least 2^{k-1} states
- $L_k = \{\ w \mid w \in \{0,1\}^*,\ k^{th}$ last symbol of w is a $1\ \}$

Stuff from last lecture

- Given DFA $M = \langle Q, \Sigma, \delta, s, F \rangle$
- Strings $x, y \in \Sigma^*$ are indistinguishable by M
  - $x \equiv_M y$ ← Equivalence relation
  - X and y end up in the same state in M (from s)



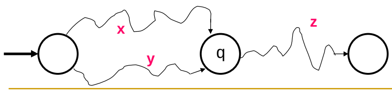


Another equivalence relation

- Given a language A over Σ
- String $x, y \in \Sigma^*$ are indistinguishable
  - $x \equiv_A y$
  - For all $z \in \Sigma^*$, $xz \in A \Leftrightarrow yz \in A$

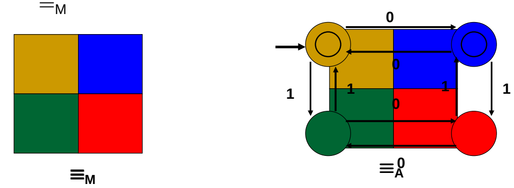
What is the relation b/w $\equiv_M$ and $\equiv_A$?

- Let M be a DFA such that A = L(M)
- If $x \equiv_M y$ then $x \equiv_A y$
- Basic intuition:
  - Once x and y have been consumed, both reach the same state (say q)
  - Thus for any z, the path followed by xz and yz after q would be the same



If $x \equiv_M y$ then $x \equiv_A y$

- Any equivalence class in $\equiv_A$ is union of equivalence classes in $\equiv_M$
  - An eqv class in $\equiv_A$ could be the same as one in $\equiv_M$



Some implications

- # eqv classes in $\equiv_A \leq$ #eqv classes in $\equiv_M$
  - For any M such that A = L(M)
- If A is regular then $\equiv_A$ has finite # of eqv classes
  - Consider any DFA M such that A=L(M)

Proving languages are not regular

- If $\equiv_A$ has infinite eqv classes then A is not regular
- What is the "method" ?
  - Find an infinite sequence of strings $x_1, x_2, \ldots$
  - For any x_i and x_j, $x_i \not\equiv_A x_j$
  - Find a string z_{ij} such that $x_i z_{ij} \in A$ but $x_j z_{ij}$ not $\in A$

Recall $x_i \equiv_A x_j$ iff for all z, $x_i z \in A$ if and only if $x_j z \in A$

An example

- $A = \{ 0^n 1^n \mid n \geq 0 \}$

- Let $x_i = 0^i$, i=1,2,…
- Need to show x_i and x_j are not equivalent
  - $z_{ij} = 1^i$
  - $x_i z_{ij} = 0^i 1^i \in A$, but
  - $x_j z_{ij} = 0^j 1^i$ not $\in A$

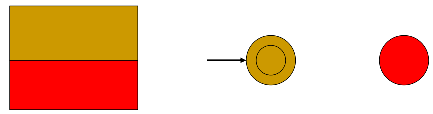
Questions ?

Myhill-Nerode theorem

- A is regular ⇔ $\equiv_A$ has finite number of eqv classes
- ⇒ we have seen
- ⇐ by construction
  - Build a DFA where each class gets its own state
  - Note that it will be the minimized DFA

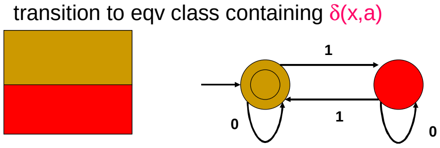
Constructing a DFA from $\equiv_A$

- One state per eqv class
- Start state: class containing ε
- Final state(s): eqv class contained in A



Constructing a DFA from $\equiv_A$

- Transitions
  - If $x \equiv_A y$ then $xa \equiv_A ya$, $a \in \Sigma$
    - For all $z \in \Sigma^*$, $xz \in A \Leftrightarrow yz \in A$
    - In particular for all $z' \in \Sigma^*$, $x(az') \in A \Leftrightarrow y(az') \in A$
  - For any eqv class and $a \in \Sigma$, pick x and send the transition to eqv class containing $\delta(x,a)$



Constructing DFA w/ minimum states

- We know the minimized DFA has number of states equal to the number of classes in $\equiv_A$
- Given $\equiv_A$, we know how to construct a DFA
- Hence, done ?

There is a catch…

Minimizing DFAs

- We might not know $\equiv_A$
- Might only know the DFA that accepts A