Using Pumping lemma for CFLs

Atri Rudra

May 19

Announcements

- Turn in your H/W #6
- Sorry, no graded H/W #5's today
  - Will be handed out in class on Monday
- Take a copy of H/W #7
- Take a copy of solutions to H/W #5
  - If you did not last class

A few words on the feedback

- Thank you
- Microphone does not work properly (?)
- Solutions to sample final
  - Will **try** and give solution sketches
- Instructor speaks too fast
- Proofs without examples are useless
  - How to use a proof ?
- Sometimes get lost in long proofs
  - What does a small step achieve in the big picture

Puzzle for today

- Prove that the following language is not a CFL
- $\{ a^n \mid n$ is a prime $\}$

Statement of the pumping lemma

- If L is a CFL then
  - $\exists$ integer $p \geq 1$
  - $\forall$ strings $s \in L$ with $|s| \geq p$
  - $\exists$ strings u,v,x,y,z satisfying s=uvxyz with
    - $|vxy| \leq p$
    - $|v| > 0$ or $|y|>0$
  - $\forall$ integer $i \geq 0$, $uv^ixy^iz \in L$

The "proof"

- L is CFL
- L is accepted by a grammar G
- Consider any string "long enough" s in L
  - The parse tree must have a repeated variable
- Repeating the derivation between the repeats will give new strings that are also in L

Contrapositive of the pumping lemma

- If L is a CFL then
  - $\exists$ integer $p \geq 1$
  - $\forall$ strings $s \in L$ with $|s| \geq p$
  - $\exists$ strings u,v,x,y,z satisfying s=uvxyz with
    - $|vxy| \leq p$
    - $|v|$ or $|y| > 0$
  - $\forall$ integer $i \geq 0$, uv^ixy^iz in L
- If
  - $\forall$ integer $p \geq 1$
  - $\exists$ string $s \in L$ with $|s| \geq p$
  - $\forall$ strings u,v,x,y,z satisfying s=uvxyz with
    - $|vxy| \leq p$
    - $|v|>0$ or $|y| > 0$
  - $\exists$ integer $i \geq 0$, uv^ixy^iz not in L
- then L is not CFL

Using the pumping lemma

- If
  - $\forall$ integer $p \geq 1$
  - $\exists$ string $s \in L$ with $|s| \geq p$
  - $\forall$ strings u,v,x,y,z satisfying s=uvxyz with
    - $|vxy| \leq p$
    - $|v|>0$ or $|y| > 0$
  - $\exists$ integer $i \geq 0$, uv^ixy^iz not in L
- then L is not CFL
- Round 1:
  - Devil picks p

- Round 2: You
  - You pick $s \in L$
- Round 3:
  - Devil picks u,v,x,y,z

- Round 4:
  - You pick i

You win if uv^ixy^iz not $\in$ L

You want to win no matter what the devil does

Let's start the duel with the devil

- $\{ a^nb^nc^n \mid n \geq 0 \}$

What have we done till now ?

- Worked with the Simpson parents

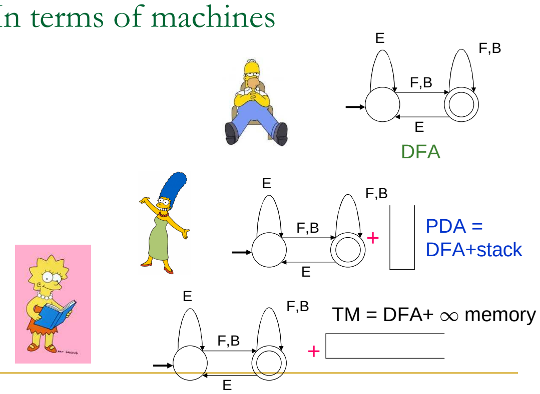
Regular languages

Context-Free languages

Now we move onto

- Turing machines (aka Lisa Simpson)

In terms of machines



A quick question

- Which model represents a PC ?
- Depends on how “faithful” representation you want
- No PC has infinite memory
- But for “practical” purposes it is “infinite”