CSE 322: The Last Homework Assignment

Due Date: Friday, March 10 (at the *beginning* of class)
Note: Finals on Monday, March 13, 2:30-4:20 pm in class

1. (20 points) Show that the language SEARCH = $\{w\#x \mid w, x \in \{0,1\}^*$ and w is a substring of $x\}$ is decidable by giving an *implementation level description* of a decider TM M for the language. (Give your TM in the format M = "On input string *s*:…"; no need to give the state diagram). See examples of implementation level descriptions in Section 3.1 in the text.

2. (20 points) Show that a language is decidable iff some enumerator enumerates the language in lexicographic order.

3. (20 points; 10 each)
   a. Show that decidable languages are closed under the concatenation operation.
   b. Show that Turing-recognizable languages are closed under the union operation

   Give the necessary Turing machines in each case using the format M = "On input *w*:…".

4. (20 points) Let $NO\text{-}INT_{\text{DFA}} = \{ \langle A,B \rangle \mid A$ and B are DFAs and $L(A) \cap L(B) = \emptyset\}$. Prove that $NO\text{-}INT_{\text{DFA}}$ is decidable. (Hint: Construct your decider TM based on one of the deciders constructed in proofs for theorems in Section 4.1. Explain your proof idea and then give your TM in the format M = "On input …: ").

5. (20 points) Let $NO\text{-}INT_{\text{TM}} = \{ \langle A,B \rangle \mid A$ and B are TMs and $L(A) \cap L(B) = \emptyset\}$. Show that $NO\text{-}INT_{\text{TM}}$ is undecidable by giving a reduction from a known undecidable language to $NO\text{-}INT_{\text{TM}}$. For your reduction, you may use any of the languages shown to be undecidable in Section 5.1 in the textbook (up to Theorem 5.4 and its proof (both editions of the text); no need to read beyond this theorem for this course).

Bonus question (no points!): Is the question "Does God exist?" decidable? (Hint: Assume the question has an unambiguous yes or no answer).