CSE 322
Intro to Formal Models in CS
Homework #4
Due: Friday 26 Oct 07
20 Oct 07

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$\boxed{\text{Midterm: Friday, Nov 2.}}$

Modified Late Policy for this assignment: It will *not* be accepted later than 5:00PM Monday, 10/29.

Text book problems below are on pages 88-91.

1. Let x and y be strings and let L be any language. We say that x and y are *equivalent with respect to* L if for every string $z \in \Sigma^*$ either xz and yz are both in L, or neither is. [So, x and y are *not* equivalent if and only if there is some string z such that exactly one of xz, yz is in L. Such a z is said to *separate* x from y with respect to L.] Suppose L is accepted by DFA M.
   (a) Prove: if M is in the same state after reading x as it is after reading y, then x and y are equivalent with respect to L.
   (b) Give an example showing that the converse of statement (1a) above is false.
   (c) Prove: if there are k strings $\{x_1, \ldots, x_k\}$ no two of which are equivalent with respect to L, then M has at least k states. [Hint: pigeon hole principle.]
   (d) In lecture I sketched a $2k+2$ state DFA accepting the language $L_k = \{a^n b^n | 1 \leq n \leq k\}$. Prove that *any* DFA accepting L_2 must have at least 6 states.
   (e) [Extra Credit:] Prove that L_k requires at least $2k+2$ states for each $k \geq 1$.
   (f) [Extra Credit:] Extend the idea in part (c) to give another approach to proving that a given language is not regular. Use it to prove that $L = \{a^n b^n | 1 \leq n\}$ is not regular.
2. 1.29b (1st ed.: not present)
3. 1.30 (1st ed.: 1.18)
4. Let $\Sigma = \{a, b\}$.
   (a) Prove that $G = \{w \in \Sigma^* | w \text{ is a palindrome}\}$ is not regular.
   (b) Prove that $F = \{w \in \Sigma^* | w \text{ is not a palindrome}\}$ is not regular. [Hint: see exercise 1.14 (1st ed.: 1.10).]
5. 1.54 (1st ed.: not present)