0.2 Write formal descriptions of the following sets.

a. The set containing the numbers 1, 10, and 100.
b. The set containing all integers that are greater than 5.
c. The set containing all natural numbers that are less than 5.
d. The set containing the string aba.
e. The set containing the empty string.
f. The set containing nothing at all.

0.3 Let A be the set $\{\mathtt{x}, \mathtt{y}, \mathtt{z}\}$, and B be the set $\{\mathtt{x}, \mathtt{y}\}$.

a. Is A a subset of B?
b. Is B a subset of A?
c. What is $A \cup B$?
d. What is $A \cap B$?
e. What is $A \times B$?
f. What is the power set of B?

0.4 If A has a elements and B has b elements, how many elements are in $A \times B$? Explain your answer.

0.5 If C is a set with c elements, how many elements are in the power set of C? Explain your answer.

0.6 Let X be the set $\{1, 2, 3, 4, 5\}$ and Y be the set $\{6, 7, 8, 9, 10\}$. The unary function $f: X \longrightarrow Y$ and the binary function $g: X \times Y \longrightarrow Y$ are described in the following tables.

n	$f(n)$
1	6
2	7
3	6
4	7
5	6

g	6	7	8	9	10
1	10	10	10	10	10
2	7	8	9	10	6
3	7	7	8	8	9
4	9	8	7	6	10
5	6	6	6	6	6

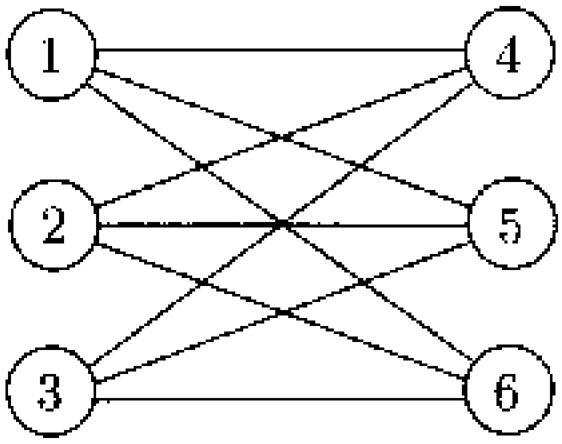
a. What is the value of $f(2)$?
b. What are the range and domain of f?
c. What is the value of $g(2, 10)$?
d. What are the range and domain of g?
e. What is the value of $g(4, f(4))$?

0.7 For each part, give a relation that satisfies the condition.

a. Reflexive and symmetric but not transitive
b. Reflexive and transitive but not symmetric
c. Symmetric and transitive but not reflexive

0.8 Consider the undirected graph $G=(V, E)$ where V, the set of nodes, is $\{1, 2, 3, 4\}$ and E, the set of edges, is $\{\{1,2\}, \{2,3\}, \{1,3\}, \{2,4\}, \{1,4\}\}$. Draw the graph G. What is the degree of node 1? of node 3? Indicate a path from node 3 to node 4 on your drawing of G.

0.9 Write a formal description of the following graph.



PROBLEMS

0.10 Find the error in the following proof that $1 = 2$.
Consider the equation $a = b$. Multiply both sides by a to obtain $a^2 = ab$. Subtract b^2 from both sides to get $a^2 - b^2 = ab - b^2$. Now factor each side, $(a+b)(a-b) = b(a-b)$, and divide each side by $(a-b)$, to get $a + b = b$. Finally, let a and b equal 1, which shows that $2 = 1$.

0.11 Find the error in the following proof that all horses are the same color.
CLAIM: In any set of h horses, all horses are the same color.
PROOF: By induction on h.

Basis: For $h = 1$. In any set containing just one horse, all horses clearly are the same color.

Induction step: For $k \geq 1$ assume that the claim is true for $h = k$ and prove that it is true for $h = k+1$. Take any set H of $k+1$ horses. We show that all the horses in this set are the same color. Remove one horse from this set to obtain the set H_1 with just k horses. By the induction hypothesis, all the horses in H_1 are the same color. Now replace the removed horse and remove a different one to obtain the set H_2. By the same argument, all the horses in H_2 are the same color. Therefore all the horses in H must be the same color, and the proof is complete.

0.12** Ramsey's theorem. Let G be a graph. A ***clique in G is a subgraph in which every two nodes are connected by an edge. An ***anti-clique***, also called an ***independent set***, is a subgraph in which every two nodes are not connected by an edge. Show that every graph with n nodes contains either a clique or an anti-clique with at least $\frac{1}{2}\log_2 n$ nodes.

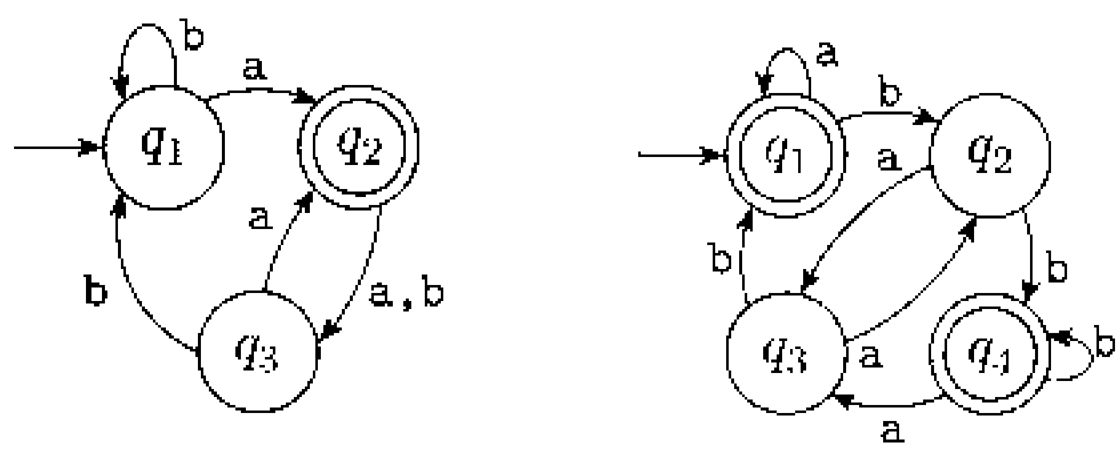
0.13 Use Theorem 0.15 to derive a formula for calculating the size of the monthly payment for a mortgage in terms of the principal P, interest rate I, and the number of payments t. Assume that, after t payments have been made, the loan amount is reduced to 0. Use the formula to calculate the dollar amount of each monthly payment for a 30-year mortgage with 360 monthly payments on an initial loan amount of \$100,000 with an 8% annual interest rate.

conditions of the pumping lemma. By condition 3, y consists only of 0s. Let's examine the string $xyyz$ to see whether it can be in E. Adding an extra copy of y increases the number of 0s. But, E contains all strings in 0^*1^* that have more 0s than 1s, so increasing the number of 0s will still give a string in E. No contradiction occurs. We need to try something else.

The pumping lemma states that $xy^iz \in E$ even when $i = 0$, so let's consider the string $xy^0z = xz$. Removing string y decreases the number of 0s in s. Recall that s has just one more 0 than 1. Therefore xz cannot not have more 0s than 1s, so it cannot be a member of E. Thus we obtain a contradiction.

EXERCISES

1.1 The following are the state diagrams of two DFAs, M_1 and M_2. Answer the following questions about these machines.



M_1 M_2

a. What is the start state of M_1?

b. What is the set of accept states of M_1?

c. What is the start state of M_2?

d. What is the set of accept states of M_2?

e. What sequence of states does M_1 go through on input aabb?

f. Does M_1 accept the string aabb?

g. Does M_2 accept the string ε?

1.2 Give the formal description of the machines M_1 and M_2 pictured in Exercise 1.1.

1.3 The formal description of a DFA M is $(\{q_1, q_2, q_3, q_4, q_5\}, \{\mathtt{u}, \mathtt{d}\}, \delta, q_3, \{q_3\})$, where δ is given by the following table. Give the state diagram of this machine.

	u	d
q_1	q_1	q_2
q_2	q_1	q_3
q_3	q_2	q_4
q_4	q_3	q_5
q_5	q_4	q_5

1.4 Give state diagrams of DFAs recognizing the following languages. In all cases the alphabet is $\{0,1\}$.

- **a.** $\{w \mid w$ begins with a 1 and ends with a 0$\}$.
- **b.** $\{w \mid w$ contains at least three 1s$\}$.
- **c.** $\{w \mid w$ contains the substring 0101, i.e., $w = x0101y$ for some x and $y\}$.
- **d.** $\{w \mid w$ has length at least 3 and its third symbol is a 0$\}$.
- **e.** $\{w \mid w$ starts with 0 and has odd length, or starts with 1 and has even length$\}$.
- **f.** $\{w \mid w$ doesn't contain the substring 110$\}$.
- **g.** $\{w \mid$ the length of w is at most 5$\}$.
- **h.** $\{w \mid w$ is any string except 11 and 111$\}$.
- **i.** $\{w \mid$ every odd position of w is a 1$\}$.
- **j.** $\{w \mid w$ contains at least two 0s and at most one 1$\}$.
- **k.** $\{\varepsilon, 0\}$.
- **l.** $\{w \mid w$ contains an even number of 0s, or exactly two 1s$\}$.
- **m.** The empty set.
- **n.** All strings except the empty string.

1.5 Give NFAs with the specified number of states recognizing each the following languages.

- **a.** The language $\{w \mid w$ ends with 00$\}$ with three states.
- **b.** The language of Exercise 1.4c with five states.
- **c.** The language of Exercise 1.4l with six states.
- **d.** The language $\{0\}$ with two states.
- **e.** The language $0^*1^*0^*0$ with three states.
- **f.** The language $\{\varepsilon\}$ with one state.
- **g.** The language 0^* with one state.

1.6 Use the construction given in the proof of Theorem 1.22 to give the state diagrams of NFAs recognizing the union of the languages described in

- **a.** Exercises 1.4a and 1.4b.
- **b.** Exercises 1.4c and 1.4f.

1.7 Use the construction given in the proof of Theorem 1.23 to give the state diagrams of NFAs recognizing the concatenation of the languages described in

- **a.** Exercises 1.4g and 1.4i.
- **b.** Exercises 1.4b and 1.4m.