

PL: if A is regular, then **there exists** p that, for **any** s in A and $|s| > p$, then **there exists** a partition $s=xyz$, satisfying condition:

1. for each $i \geq 0$, $xy^i z$ in A
2. $|y| > 0$
3. $|xy| < p$

PL \Rightarrow all regular languages are infinite

F All finite languages are regular

Every DFA contains a loop

T DFA runs on input of arbitrary length, there must be a loop

Every DFA contains a loop from which a final state is reachable

F excludes DFAs for finite languages.

$L = \{a^n b^n \mid n \geq 0\}$ is not regular

T pumping lemma

Any subset of that L is not regular

F empty subset

An infinite subset of that L is not regular

T pumping lemma

if that L is a subset of L', then L' is not regular

F Σ^*

if L1 union L2 is regular then so are L1 and L2

F $L1 \text{ union } L2 = \Sigma^*$

if L_1 intersection L_2 is regular then so are L_1 and L_2

F L_1 and L_2 disjoint

If L_1 and L_2 are regular, then L_1 union L_2 is regular

T closure property

If L_1 and L_2 are regular, then L_1 intersection L_2 is regular

T closure property

Application of Pumping Lemma.

$$\Sigma = \{0, 1, +, =\}$$

$ADD = \{a=b+c \mid a, b, c \text{ are binary integers and } a \text{ is sum of } b \text{ and } c\}$.

Solution:

$$a=10^p, b=1^p, c=1$$

$$|xy| < p \text{ and } |y| > 1 \Rightarrow x=\varepsilon \quad y=10^i \quad \text{or} \quad x=10^i \quad y=0^j$$

Proof by closure properties of regular expression

If L intersects L' (L' is regular) is not regular, then L is not regular.

$$\Sigma = \{0, 1\}, \quad L = \{\text{the number of } 0\text{'s and the number of } 1\text{'s are equal}\}$$

$$L \text{ intersects } L' = \{0^i 1^j \mid i, j \geq 0\} = \{0^i 1^i \mid i \geq 0\}$$

L' is regular, and L intersects L' is not regular $\Rightarrow L$ is not regular.

Many elements of programming languages are regular, e.g.

Identifiers: the first being a letter of the alphabet or an underline,
and the remaining being any letter of the alphabet, any numeric
digit, or the underline

int/float

keywords.

A C program is not regular.

```
main(){return (...(0)...);}
```