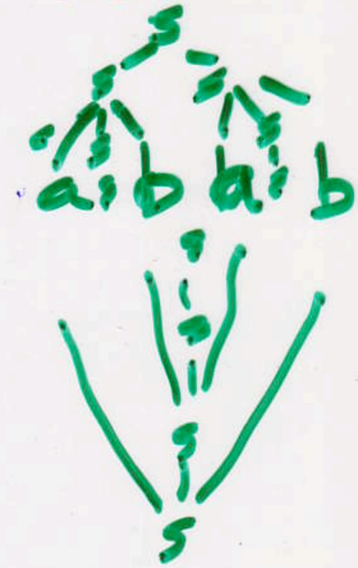
$S \to \varepsilon \mid aSb \mid bSa \mid SS$

$L(G) = \{ w \in \Sigma^* \mid \#_a(w) = \#_b(w) \}$ (*)

if $S \Rightarrow^* w \in \Sigma^*$ then w

→ if w satisfies (*) then $S \Rightarrow^* w$ (**)



induction on $|w|$

base $|w| = 0$

$w = \varepsilon$

$S \Rightarrow^* w$

Ind: suppose $|w| > 0$

$\#_a(w) = \#_b(w)$ &

(**) true for all ~~⇒~~ shorter strings

$S \Rightarrow^* w$

case1 if $w = aw'b$ then w' is

(a) shorter than w &

(b) w' satisfies (*)

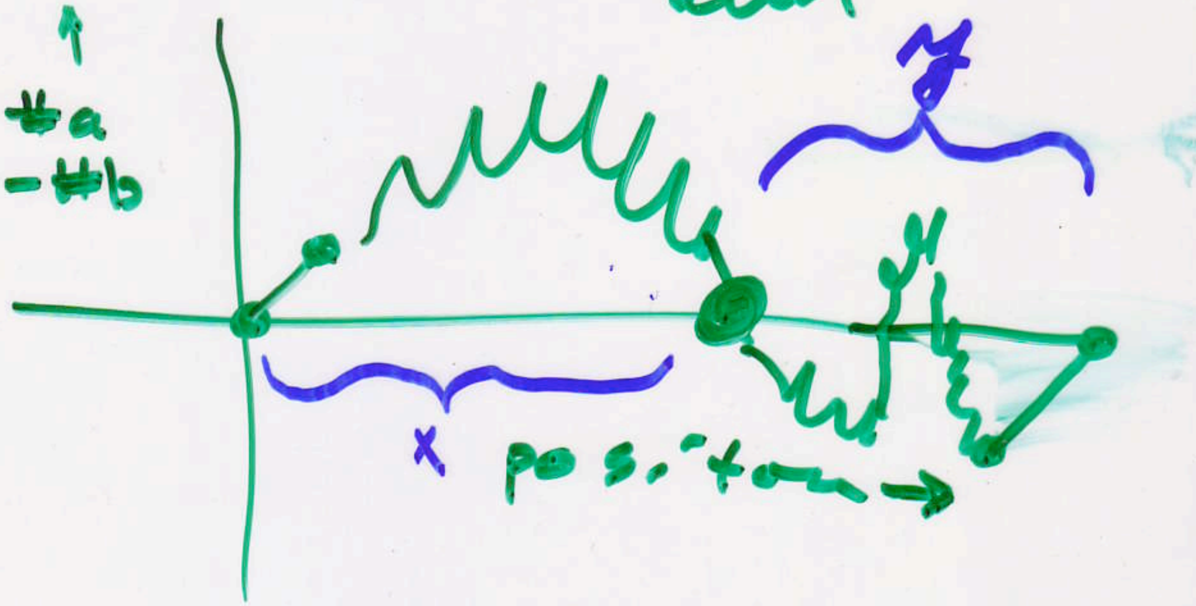
By I.H. $S \Rightarrow^* w'$

$S \Rightarrow aSb \Rightarrow^* aw'b = w$

Case 2 $w = bw'a$

similar

Case 3 w starts & end w/ same letter



Must be x, y st

$w = xy$, x, y both satisfy ✱

neither is empty

By I. H.

$S \overset{*}{\Rightarrow} x$

$S \overset{*}{\Rightarrow} y$

$\therefore S \Rightarrow SS \overset{*}{\Rightarrow} xS \overset{*}{\Rightarrow} xy = w$

$S \to aSbS \mid bSaS \mid \varepsilon$

$S \to (S)S \mid \varepsilon$

$L = \{ ww^R \mid w \in \{a,b\}^* \}$

$S \to aSa \mid bSb \mid \varepsilon$

$L_2 = \{ ww \mid w \in \{a,b\}^* \}$

L_2 is ~~not~~ CFL

$S \to SaSa \mid SbSb$ ✗

$S \to TT$
$T \to \varepsilon \mid aT \mid bT$ ✗

$S \to TT \mid PP \mid \varepsilon$
$T \to aT \quad \ldots$
$P \to bP \quad \ldots$ ✗

$\{ a^n b^n \mid n \geq 0 \}$

$S \to aSb \mid \varepsilon$

$\{ a^n b^n c^n \mid n \geq 0 \}$

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if L is regular
then L is a CFL.