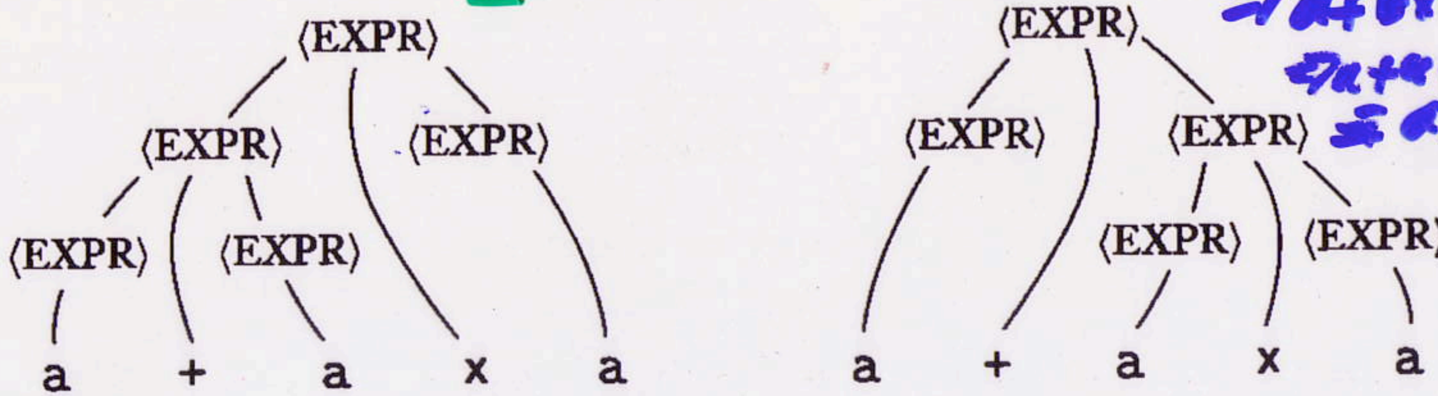
$E \to E + E \mid E * E \mid a$

ambiguous $\neq$

$E \Rightarrow E+E$
$\Rightarrow a+E$
$\Rightarrow a+E*E$
$\Rightarrow a+a*E$
$\Rightarrow a+a*a$



3 + 4 * 5

FIGURE 2.6
The two parse trees for the string a+a×a in grammar G_5

Leftmost deriv

$E \Rightarrow_L E * E \Rightarrow_L E+E*E \Rightarrow_L a+E*E$
$\Rightarrow_L a+a*E$
$\Rightarrow_L a+a*a$

EXAMPLE 2.4 ....

Consider grammar $G_4 = (V, \Sigma, R, \langle \text{EXPR} \rangle)$.

V is $\{\langle \text{EXPR} \rangle, \langle \text{TERM} \rangle, \langle \text{FACTOR} \rangle\}$ and Σ is $\{\texttt{a}, \texttt{+}, \texttt{x}, \texttt{(}, \texttt{)}\}$. The rules are

unambig G for same L.

$$\langle \text{EXPR} \rangle \to \langle \text{EXPR} \rangle\texttt{+}\langle \text{TERM} \rangle \mid \langle \text{TERM} \rangle$$
$$\langle \text{TERM} \rangle \to \langle \text{TERM} \rangle\texttt{x}\langle \text{FACTOR} \rangle \mid \langle \text{FACTOR} \rangle$$
$$\langle \text{FACTOR} \rangle \to \texttt{(}\,\langle \text{EXPR} \rangle\,\texttt{)} \mid \texttt{a}$$

The two strings a+axa and (a+a)xa can be generated with grammar G_4. The parse trees are shown in the following figure.

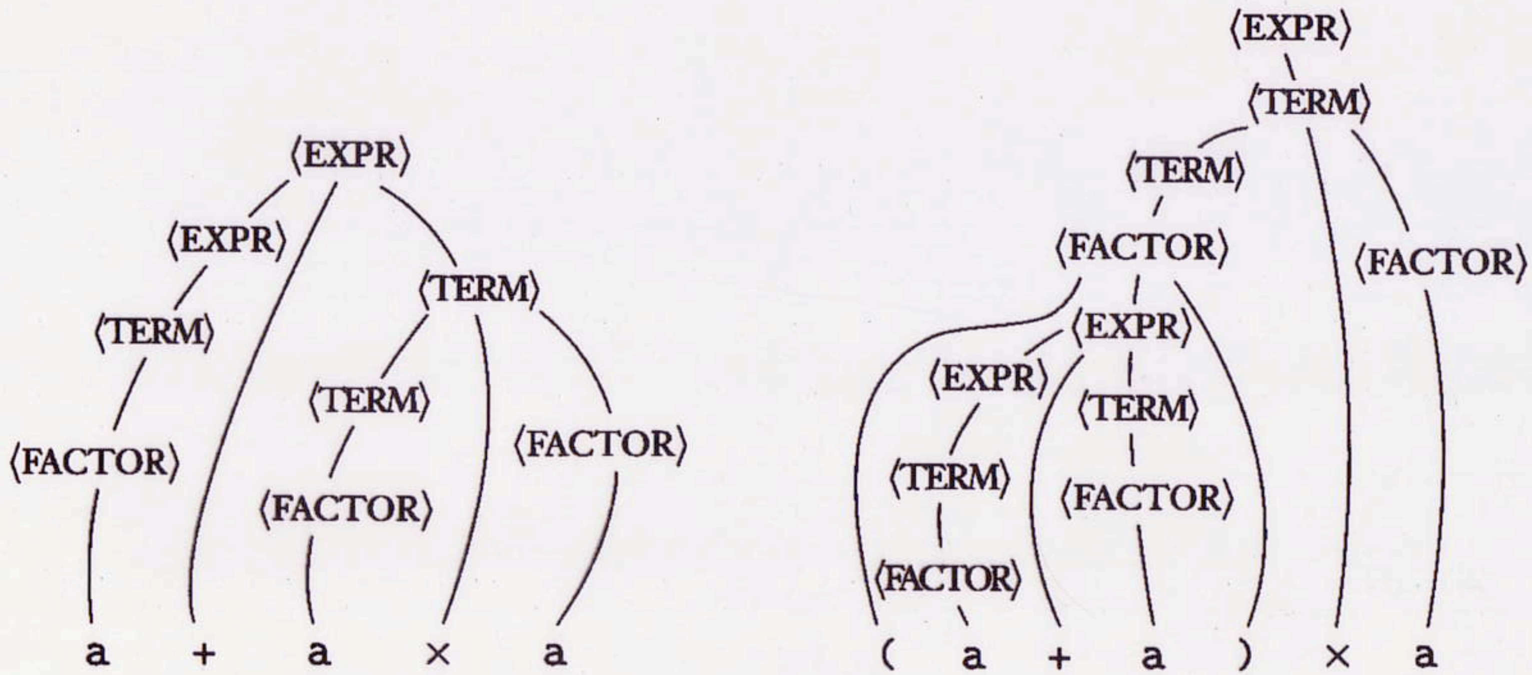


FIGURE 2.5

Parse trees for the strings a+axa and (a+a)xa

20-2

$\{ a^i b^j c^k \mid i = j \text{ or } j = k \}$

$a^n b^n c^n$

inherently ambiguous

if L_1 & L_2 are CFL then
$L_1 \cdot L_2$ is a CFL.

$G_i = (V_i, \Sigma, R_i, S_i)$ $i = 1, 2$

with $L_i = L(G_i)$

Assume $V_1 \cap V_2 = \emptyset$ & $S \notin V_1 \cup V_2$

$G_3 = (V_1 \cup V_2 \cup \{S\}, \Sigma, R_3, S)$

$R_3 = R_1 \cup R_2 \cup \{S \to S_1 S_2\}$

if $x \in L_1$ & $y \in L_2$
Then $xy \in L(G_3)$ OK

$S \Rightarrow_L S_1 S_2 \Rightarrow_L^* x S_2 \Rightarrow_L^* xy$

if $w \in L(G_3)$ then $\exists x, y$ st
$x \in L_1$ $y \in L_2$ & $w = xy$.

$S \Rightarrow^* w$

in G_3 [$S \Rightarrow S_1 S_2 \Rightarrow_L^* x S_2 \Rightarrow_L^* xy$

so $S_1 \Rightarrow^* x$ in G_1 & $S_2 \Rightarrow^* y$ in G_2

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