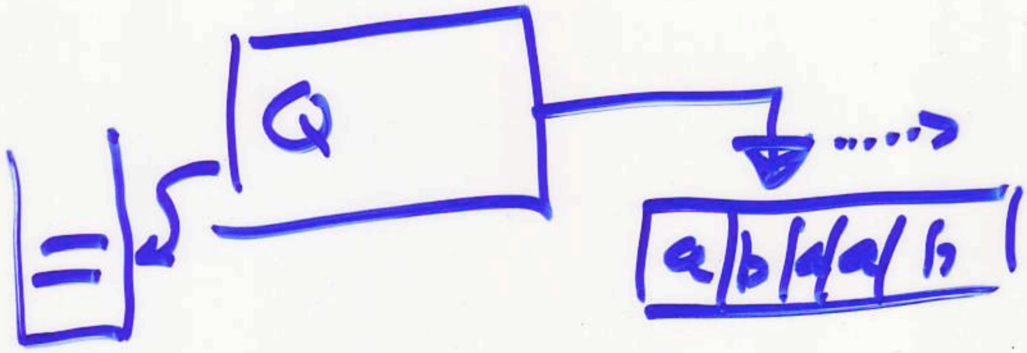
CF but not Regular

$a^n b^n$, ww^R, ...



Recursive

$S \to aSb \mid \varepsilon$

$S \to aSa \mid bSb \mid \varepsilon$

Intuit

$a^n b^n$: push a's
pop/match b's

ww^R : push input
guess when reach middle
pop/match

Pushdown Automaton

$M = (Q, \Sigma, \Gamma, \delta, q_0, F)$

Q is finite set (states)

Σ . . . . . . (input alphabet)

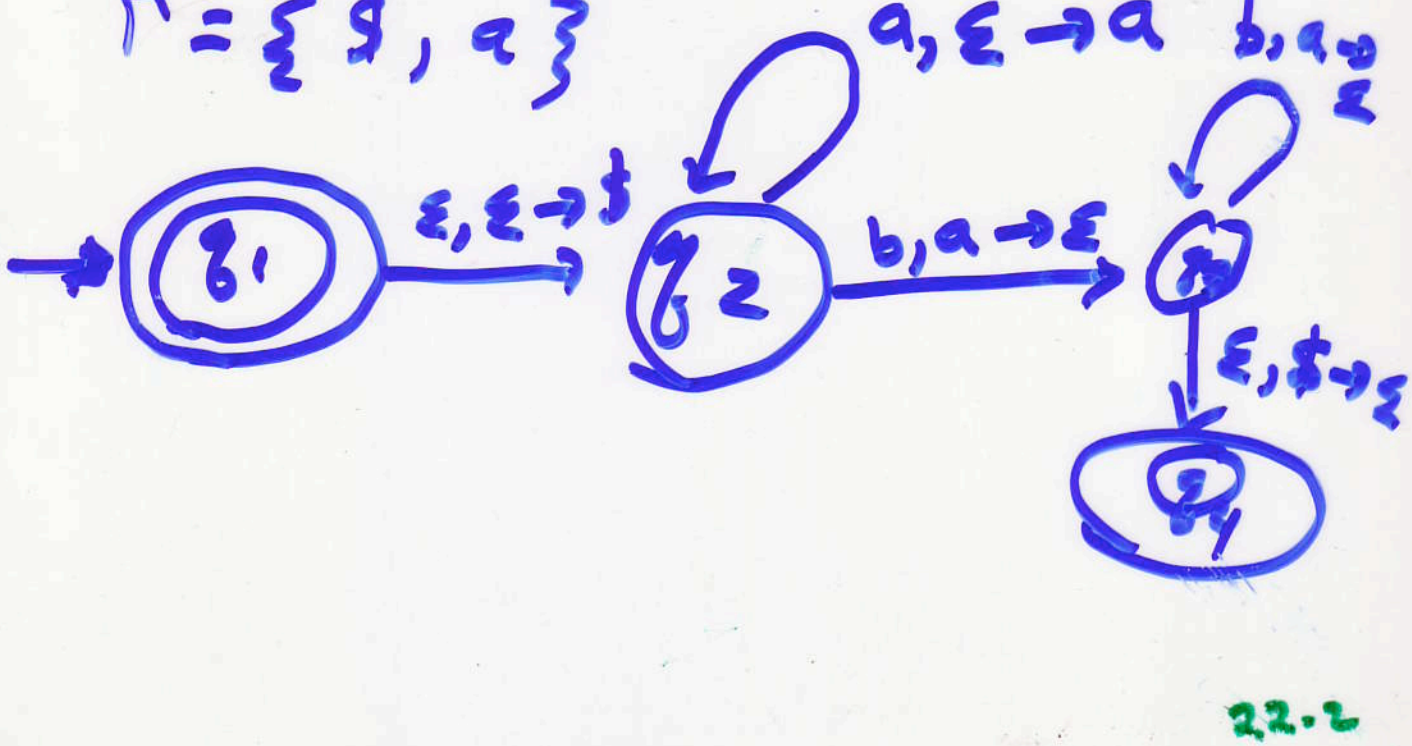
Γ . . . . . . (Stack alphabet)

$q_0 \in Q$ start state

$F \subseteq Q$ ~~Final state~~ accepting

$$\delta: Q \times \Sigma_\varepsilon \times \Gamma_\varepsilon \to 2^{Q \times \Gamma_\varepsilon}$$

$\Gamma = \{\$, a\}$



M can reach state q with $\gamma \in \Gamma^*$ on its stack if

$\exists\, w_1 w_2 \dots w_m \in \Sigma_\varepsilon$

st $w = w_1 \cdot w_2 \dots w_m$

$\exists\, r_0 r_1 \dots r_m \in Q$

$\exists\, s_0, \dots s_m \in \Gamma^*$

(1) $r_0 = q_0$,

(2) $s_0 = \varepsilon$

(3) $\forall\, i = 0 .. m-1$

$(r_{i+1}, b) \in \delta(r_i, \underbrace{w_{i+1}}, a)$

for some $a, b \in \Gamma_\varepsilon$

with $s_i = a\,t$, $s_{i+1} = b\,t$

and $t \in \Gamma^*$

M accepts w if $r_m \in F$

Example: A computation of M above on input $w = aabb$

State	Stack	remaining input	
$r_0 = q_1$	$s_0 = \varepsilon$	$aabb$	$(q_2, \$) \in \delta(q_1, \varepsilon, \varepsilon)$
$r_1 = q_2$	$s_1 = \$$	$aabb$	$(q_2, a) \in \delta(q_2, a, \varepsilon)$
$r_2 = q_2$	$s_2 = \$a$	abb	" "
$r_3 = q_2$	$s_3 = \$aa$	bb	$(q_3, \varepsilon) \in \delta(q_2, b, a)$
$r_4 = q_3$	$s_4 = \$a$	b	$(q_3, \varepsilon) \in \delta(q_3, b, a)$
$r_5 = q_3$	$s_5 = \$$	ε	$(q_4, \varepsilon) \in \delta(q_3, \varepsilon, \$)$
$r_6 = q_4$	$s_6 = \varepsilon$	ε	

top of stack @ right end

So, e.g., "M can reach q_3 w/ \$ on stack reading a^2b" and "M can reach q_4 w/ ε stack reading a^2b^2" and "M accepts a^2b^2".