Reading Assignment: DFA minimization notes, String matching notes, and Sipser 2.1

1. Consider the DFA $M=\left(Q, \Sigma, \delta, q_{0}, F\right)$ with $Q=\{A, B, C, D, E, F, G, H\}, \Sigma=\{0,1\}$, $q_{0}=A, F=\{D\}$ and the transition function given by the transition table:

|  | 0 | 1 |
| :--- | :--- | :--- |
| A | B | A |
| B | A | C |
| C | D | B |
| D | D | A |
| E | D | F |
| F | G | E |
| G | F | G |
| H | G | D |

Construct the minimum-state equivalent DFA for $M$.
2. Design context-free grammars which generate each of the following languages. Justify your grammar designs. You may specify your CFG by writing out the rules with the convention that the first state is the start state. You may also use the "" shorthand.
(a) The set $\left\{w \in\{0,1\}^{*} \mid w=w^{R}\right\}$, i.e. the set of palindromic binary strings.
(b) The complement of the set $\left\{a^{n} b^{n} \mid n \geq 0\right\}$ in $\{a, b\}^{*}$.
(c) The set $\left\{w \in\{0,1\}^{*} \mid\right.$ the length of $w$ is odd and its middle symbol is 0$\}$.
3. Consider the grammar:

$$
S \rightarrow(S)|S S| \varepsilon
$$

(a) Show that this grammar is ambiguous.
(b) Give a new unambiguous grammar for the same language.
4. Extra credit: Do it for the glory not the points. Let the alphabet be $\Sigma=\{a, b\}$. Give a CFG generating the language of strings with twice as many as as $b s$. Prove that your grammar is correct.

