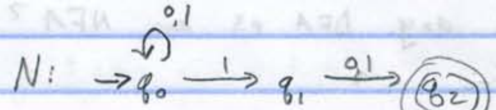
4/19/2010

For HW #3

\# 1) Should be an NFA, not a DFA

\#2) Don't write down all the states for the DFA, just those necessary for the transitions

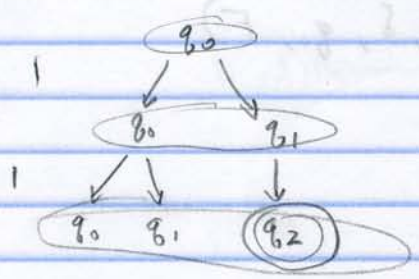
NFA



$L(N) = \{ w \mid w = x1a, \; x \in \{0,1\}^*, \; a \in \{0,1\} \}$

EXAMPLE

$w = 11$



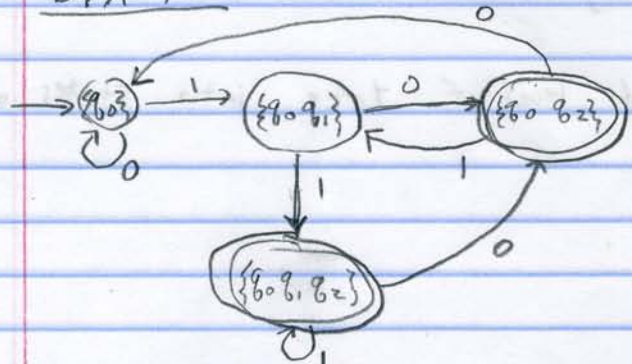
Keep track of which subsets of states we're in at any one time

(This is specific to string "11" though)

Accept 11

Convert to DFA: (Collapse the tree into a chain of states):

DFA M



$\{q_1, q_2\}$ is not reachable, so we can ignore it.

In general, many subsets will never be used.

Any state that includes q_2 is an accept state.

THM For every NFA N, $\exists$ equivalent DFA M.
s.t. $L(N) = L(M)$

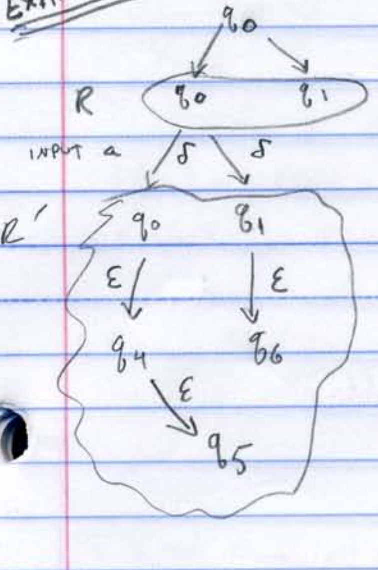
Pf: Given NFA $N = (Q, \Sigma, \delta, q_0, F)$
NFA has k states
How many possible subsets of states? 2^k
So DFA has 2^k possible states.

DFA $M = (Q', \Sigma, \delta', q_0', F')$

1. $Q' = 2^Q$
2. $\Sigma = \Sigma$
3. $q_0' = \underline{E(\{q_0\})}$
4. $F' = \{R \in Q' \mid \exists q \in R \text{ s.t. } q \in F\}$
5. $\delta'(R, a) = \{q' \mid q' \in \underline{E(\delta(q, a))} \text{ for } q \in R\}$

ε-closure of Set A:
$E(A) = \{q \mid q$ can be reached from a state in A via 0 or more ε transitions$\}$

EXAMPLE



COR. L is regular $\iff \exists$ DFA M, $L(M) = L$

$\iff \exists$ NFA N, $L(N) = L$

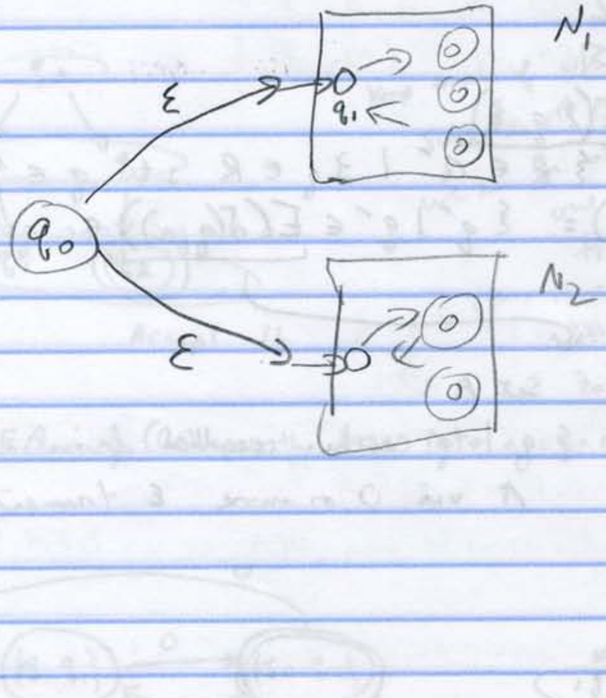
THM If A_1, A_2 are regular, $A_1 \cup A_2$ is regular

(Already proved via cartesian product construction)
Now we prove with NFA's.

Pf NFA's N_1, N_2 for A_1, A_2

NFA for

$A_1 \cup A_2$



THM A_1, A_2 Reg $\Rightarrow A_1 \circ A_2$ is regular

Pf