Notes for Friday, April 23rd

To keep in mind:

DFA:

$q_i \xrightarrow{a} q_j$

maps a single state to a single state

NFA:

$q_i \xrightarrow{a} \{\_, \_, \_\}$

maps a single state to a set of states

NFA $\rightarrow$ DFA:

DFA $= (Q, \Sigma, \delta, q_0, F)$

$Q = \text{Pow}(Q')$

Transition function seems like it's going from a set of states, but it's just notation. It may be useful to imagine quotes:

"$\{q_1, q_2\}$" $\rightarrow$ "$\{\quad\}$".

We have learned two ways to describe languages: DFAs and NFAs (and we have proved they are the same).

Last time, we proved languages are closed under $\cup, \circ, *$.

Suppose we use $\cup, \circ, *$ to describe languages. Some examples (we use $\Sigma = \{0, 1\} = 0 \cup 1$.):

1. $0 \cup 1 = \{0, 1\}$.
2. $(0 \cup 1) \circ 0 = \{00, 10\}$
3. $(0 \cup 1)^* = \{0, 1\}^*$ (like Σ^*)
4. $(0 \cup 1)^* 0 = \{w | w \text{ ends in } 0\}$
5. $((0 \cup 1)(0 \cup 1))^* = \{w || w | \text{ is even }\}$
6. $\Sigma^* 1 \Sigma = \{w | \text{ second to last symbol of } w \text{ is } 1\}$
7. $\Sigma^* \emptyset = \{w | \exists x, y \text{ such that } w = xy \text{ and } x \in \Sigma^*, y \in \emptyset\} = \emptyset$
8. $A\emptyset = \emptyset$
9. $\emptyset^* = \{\varepsilon\}$ (because k can be 0 in the definition of $*$)
10. $\varepsilon^* = \{\varepsilon\}$

Sets of strings described by these operations are called *Regular Expressions.*

Definition: R is a regular expression IFF

R is a string over the alphabet $\Sigma \cup \{(,), \varepsilon, \emptyset, \cup, *\}$ (we often omit $\circ$ because we may write ab instead of $a \circ b$)

AND

R is

1. $a \in \Sigma$ OR
2. ε OR
3. $\emptyset$ OR
4. $R_1 \cup R_2$, with R_1, R_2 regular expressions OR
5. $R_1 R_2$, with R_1, R_2 regular expressions OR
6. R_1^*, with R_1 a regular expression

Parentheses are used for precedence. Without them, $* > \circ > \cup$.
The language of a regular expression, $L(R)$, is the set of strings defined by R.
Examples:

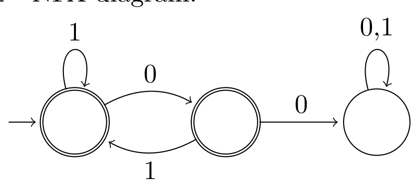
1. $L(R) = \{w | w \text{ contains exactly 2 0's}\}$:
   $R = 1^*01^*01^*$

2. $L(R) = \{w | w \text{ contains at least 2 0's}\}$:
   $R = \Sigma^*0\Sigma^*0\Sigma^*$

3. $L(R) = \{w | w \text{ contains even number of 0's}\}$:
   $R = 1^*(1^*01^*01^*)$ or $1^*(01^*01^*)$

4. $L(R) = \{w | w \text{ does not contain } 00\}$
   Consider the "opposite": $L(R') = \{w | w \text{ contains } 00\}$:
   $R' = \Sigma^*00\Sigma^*$
   Ideally, we'd like $R = \Sigma^* - \Sigma^*00\Sigma^*$, but this is not allowed.
   It may help to make a DFA for R :

   1 NFA diagram:

   What does not containg 00 mean?
   Answer: any 0 must be followed by 1, unless it is the final 0
   $(011^*)^*$ or $(011^*)^*0$
   we are still missing the 1^* case:
   $(1^*(011^*)^*) \cup (1^*(011^*)^*0)$
   or alternatively $1^*(011^*)^*(\varepsilon \cup 0)$
   The regular expression seems to capture the dynamics of the computation done by the DFA.

Question: are regular expressions and DFAs/NFAs equivalent?
Final example: $L(R) = \{w | w \text{ is a valid identifier in C}\}$
$R = (A \cup B \cup \cdots \cup Z \cup a \cup b \cup \cdots \cup z \cup \_)(A \cup B \cup \cdots \cup Z \cup a \cup b \cup \cdots \cup z \cup \_ \cup 0 \cup 1 \cup \cdots \cup 9)^*$.
Regular expressions are useful to describe the general rules.