5/15/2010

Midterm Questions

1) Are Reg Langs closed under $\subseteq$?

A Reg $\Rightarrow \forall B \quad B \subseteq A \quad B$ is reg ?

Prove or give counterexample

Counterexample: $A = \Sigma^*$ (REGULAR) $\Sigma = \{0, 1\}$

Let $B = \{0^n 1^n \mid n \geq 0\}$ = NOT REGULAR

$B \subseteq A$

2) $\{0^n 0^m 1^k \mid n, m, k \geq 0$ AND $m = 2n\}$ Is it Regular?

$= \{0^{3n} 1^k \mid n, k \geq 0\} = (000)^* 1^*$ (Reg. EXPR.)

YES, IT'S REGULAR

What is another way that we can describe non-regular languages?

Ex $B = \{0^n 1^n \mid n \geq 0\}$

$n=0$	ε
1	01
2	0011
3	000111
4	00001111

each string built from previous one by adding a 0 to left end and a 1 to right end.

string $x \in B \iff x = \varepsilon$ or $x = 0y1$ where $y \in B$

Grammar G1

$S \to \varepsilon$
$S \to 0S1$ } RULES

Any string in the language ^B can be produced using those 2 rules

"S produces ....."
$S \to \varepsilon \mid 0S1$
↑ "OR"

} Alternate ways to write rules

EX

$x = 000111$ try to generate x from the grammar

"DERIVATION OF w = 000111 in G1"

$\overbrace{S \Rightarrow 0S1 \Rightarrow 00S11 \Rightarrow 000S111}^{\text{using } S \to 0S1}$

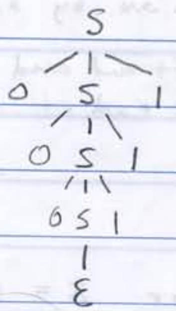
$\underbrace{\Rightarrow}_{\text{using } S \to \varepsilon} 000111$

$S \overset{*}{\Rightarrow} 000111$ (means G1 generates 000111)

$L(G_1) = \{ w \mid S \overset{*}{\Rightarrow} w \}$

$L(G_1)$ is the language generated by G1

Parse tree for 000111 in G1



Read the generated string from left-most leaf to right-most leaf.

Leaves = $000\varepsilon111$

G_2

$S \rightarrow A0A0A$

$A \rightarrow 1A \mid \varepsilon$

$L(G_2) = \{w \mid w$ contains exactly two zeroes$\}$

$= 1^*01^*01^*$

$L(G_3) = \{w \mid w$ contains at least 2 zeroes$\}$

$G3$

$S \rightarrow A0A0A$

$A \rightarrow 1A \mid 0A \mid \varepsilon$

$L(G_3) = \Sigma^*0\Sigma^*0\Sigma^*$

$L(G_4) = \{w \mid w$ contains an even number of 0's$\}$

$G4$

$S \rightarrow A0A0AS \mid A$

$A \rightarrow 1A \mid \varepsilon$

$L(G4) = 1^* \cup (1^*01^*01^*)^*$

$L(G5) = \{w \mid w \in \{0,1\}^*$ and $w = w^R\}$ (PALINDROMES)

$G5$

Easiest thing is to write down the strings to see the recursion:

ε	0	1
00	000	010
11	101	111

WE SEE THAT WE NEED TO ADD A 0 to both ENDS or a 1 to both ENDS

G5

$S \to 0S0 \mid 1S1 \mid 0 \mid 1 \mid \varepsilon$

Formal Definition of a Grammar

Context Free Grammar (CFG)

$G = (V, \Sigma, R, S)$

1. FINITE SET OF Variables $= V$ (Symbols)
2. FINITE SET OF Terminals $= \Sigma$ (ALPHABET) (DOES NOT CONTAIN ε)
3. Rules $= R$: $V \to$ STRING OVER $(V \cup \Sigma)$
   $V \to (V \cup \Sigma)^*$ Gives us ε
4. Start Symbol $S \in V$