CSE 322 Spring 2010

<u>Homework Assignment # 1</u>

Due Date: Friday, April 9 (at the *beginning* of class)

Note: N = set of natural numbers = {1, 2, 3, …}, Z = set of integers = {…, -1, 0, 1, …}

1. (20 points) Write formal descriptions of the following sets:
   Examples: Set containing 1, 10, 100 = {1, 10, 100}
   Set containing all even integers = $\{n \mid n = 2m \text{ for some } m \in Z\}$
   a. The set containing all prime numbers less than 8
   b. The set containing all odd integers greater than 100
   c. The set containing all strings over $\Sigma = \{0,1\}$ that begin with 0 and end in 1
   d. The set containing all strings over $\Sigma = \{0,1\}$ of odd length whose middle symbol is 0
   e. The set of all strings over $\Sigma = \{a, b, \dots, z\}$ that contain the word "huskies" as a substring but not the word "cougars"

2. (20 points) Let $A = \{n \mid n = 2^a \text{ for some } a \in N \text{ and } a < 3\}$ and let $B = \{n \mid n = 2m \text{ for some } m \in N \text{ and } 1 < m < 4\}$.
   a. Which of the following statements is/are true: $A \subseteq B$, $B \subseteq A$, $A \cap B \neq \varnothing$, $A \cup B = \{n \mid n = 2m \text{ for some } m \in Z \text{ and } m < 4\}$
   b. What is the complement of $A \cup B$? (Write a formal description in the form: $\{n \mid \dots\}$. Take complement with respect to N)
   c. Prove or disprove: $A–B = B–A$ (Recall: $B–A = B \cap \overline{A}$)
   d. Prove or disprove: $A \times B = B \times A$
   e. What is the power set of $A \cup B$? (List all the elements)

3. (20 points) Prove that for any three sets A, B, and C, if $A \cap B = \varnothing$ and $C \subseteq B$, then $A \cap C = \varnothing$ (Hint: Prove by contradiction)

4. (20 points) Prove, using the definition of set equality, that for all sets A and B: $(A \cap B) \cup (A-B) = A$.

5. (20 points) Your boss at Gullgoo.com isn't convinced the code fragment you wrote is correct. You decide to put your 322 knowledge to good use (since your job is on the line). Prove, using induction, that given any `x`, the following code fragment you wrote prints out $(x+n)^2$ for any natural number `n` (`sqrt` computes the square root of a number):

```
y = x*x;
for i = 1 to n
   y = y + 2*sqrt(y) + 1;
print(y);
```