CSE 322 Spring 2010

<u>Homework Assignment # 2</u>

Due Date: Friday, April 16 (at the *beginning* of class)

1. (20 points) Give examples of each of the following if possible. If not possible, explain why.
   a. Two countably infinite sets A and B such that A is a proper subset of B
   b. Two countably infinite sets whose cross product is uncountably infinite
   c. Two uncountably infinite sets whose intersection is finite
   d. Two uncountably infinite sets A and B such that (A-B) is uncountably infinite

2. (10 points) You are in the restroom of your local theatre that is playing the new disaster movie (or disaster of a movie) starring Ben Affleck. The restroom contains 6 stalls in a row. If 4 of these stalls are empty, prove that there is at least one empty stall that has another empty stall next to it. (Hint: Use the pigeonhole principle.)

3. (20 points) Consider the set Σ^* for $\Sigma = \{0,1\}$.
   a. Prove that Σ^* is countably infinite.
   b. At the annual CSE 322 theorem-proving cocktail party, a party crasher announces the following "proof" by diagonalization that Σ^* is in fact uncountable. What is wrong with this "proof"?
   "Proof: By Contradiction. Suppose Σ^* is countably infinite. Then, there exists a bijection f: $N \rightarrow \Sigma^*$. We can create a new string s as follows:
   *i*th symbol of s = 0 if the *i*th symbol of f(*i*) is 1
   1 if the *i*th symbol of f(*i*) is 0
   1 if length of f(*i*) < *i* (i.e. *i*th symbol does not exist)
   Then, s differs from all the strings given by the function f. Since s is a binary string that is not among the outputs of f, this contradicts the fact that f is a bijection. Therefore, Σ^* is uncountable."

4. (50 points) Draw state diagrams of (deterministic) finite automata that recognize the following languages. In all cases, the alphabet is {0,1}.
   a. {w | w begins with 1 and ends in 0}
   b. {w | number of 1's in w is divisible by 3}
   c. {w | the third symbol of w is 1 and w has odd length}
   d. {w | each 1 in w is immediately preceded by a 0}
   e. {w | w contains an odd number of 0s and at least two 1s}
   f. {w | w contains a single 00 and a single 11}
   g. {w | w contains at least two 0s and at most four 1s}
   h. {w | w does not contain 101 or 111}
   i. the set $\{\varepsilon\}$
   j. the empty set