Name: ______________________________________
Student ID: ______________________________________

CSE 322 Autumn 2001: Midterm Exam

(closed book, closed notes except for 1-page summary)
Total: 100 points, 5 questions, 20 points each. Time: 50 minutes

Instructions:

1. Write your name and student ID on each sheet. Write or mark your answers in the space provided. If you need more space or scratch paper, you can get additional sheets from the instructor. Make sure you write down the question number and your name/id on any additional sheets.
2. Read all questions carefully before answering them. Feel free to come to the front to ask for clarifications.
3. *Hint 1*: You may answer the questions in any order, so if you find that you're having trouble with one of them, move on to another one that seems easier.
4. *Hint 2*: If you don't know the answer to a question, don't omit it - do the best you can! You may still get partial credit for whatever you wrote down. Good luck!

1. (20 points) Circle True (T) or False (F) below. Very briefly justify your answers (e.g. by giving an example or counter-example, by citing a theorem or result we proved in class, or by *briefly* sketching a construction).

a. Any infinite subset of an uncountably infinite set is also uncountable.…… T F
Why? **False.**
N is a infinite subset of R. R is uncountably infinite. N is not.

b. For any set A, there is always a one-to-one function from A to the power set of A……………………………………………………………………. T F
Why? **True.**
The function f(x) = {x} is one-to-one and maps an element of A to an element of the power set of A. Note: On the other hand, there cannot be an onto function (or a bijection) because |Power(A)|=$2^{|A|}$ > |A|.

c. If R is a regular language but R $\cup$ S is not regular for some language S, then S cannot be regular…………………………………………….…..…… T F
Why? **True.**
Proof by Contradiction. Given R is regular and R $\cup$ S is not. Suppose S is regular. This implies R $\cup$ S is regular (Regular languages are closed under union) which is a contradiction.

d. For any language R, if R* is regular, then R is regular…………………… T F
Why? **False.**

Take R=$0^n1^n \cup \{0,1\}$. This language is clearly not regular but $R^*=\Sigma^*$ is regular. (Other counter-examples can also be given)

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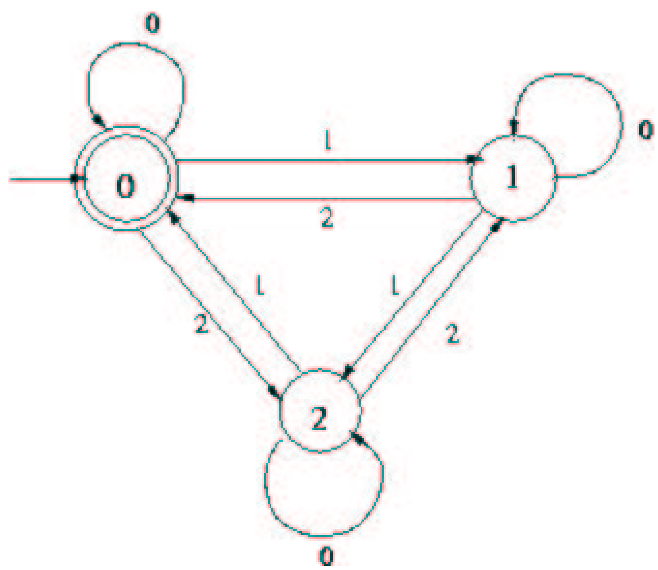
2. (20 points) DFAs

a. Let $\Sigma = \{0,1,2\}$. Draw the state diagram of a *deterministic* finite automaton (DFA) that recognizes the language $L = \{w \in \Sigma^* \mid \text{sum of } w\text{'s digits is divisible by } 3\}$. For example: 111, 012, 222, 00, ε, 21120 are all in L but 020, 10, and 211 are not.



b. Fill in the following proof for showing that the class of regular languages is closed under complement.
Proof: Let L be a regular language. Then, there exists a DFA $M = (Q, \Sigma, \delta, q_0, F)$ such that $L = L(M)$. The complement of L is recognized by the DFA $M' = (Q', \Sigma, \delta', q_0', F')$, where:

$Q' = \mathbf{Q}$

$\delta' = \boldsymbol{\delta}$

$q_0' = \mathbf{q_0}$

$F' = \mathbf{Q\text{-}F}$

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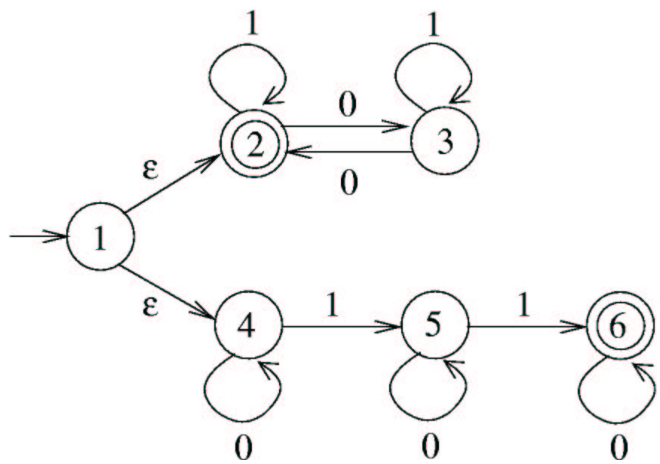
3. (20 points) NFAs

Consider the NFA N = (Q, Σ, δ, q_0, F) with the following state diagram:



a. What states can N be in after reading:
the string 0? **3, 4** the string 01? **3, 5** the string 0111? **3**

b. Does N accept 0111? **No.** Why or why not? **3 ∉ F (3 is not an accept state)**

Consider the equivalent DFA M = (Q', Σ, δ', q_0', F') derived using the "subset construction" method for converting an NFA to a DFA.

c. What is q_0' (in terms of the states of N)? q_0' = **{1,2,4}**

d. δ'({2,4},1) = **{2,5}** ? δ'({6},1) = ∅ ? δ'({3,5,6},1) = **{3,6}** ?

e. What state q ∈ Q' is M in after reading the string 00100? q = **{2,5}**

f. Is q above in F'? **Yes.** Why or why not? **Because {2,5} ∈ F' (because 2 is an accept state of N)**

g. Complete the following: L(N) = {*w* | *w* ∈ {0,1}* and ***w* has even number of 0's OR has *exactly* two 1's**}

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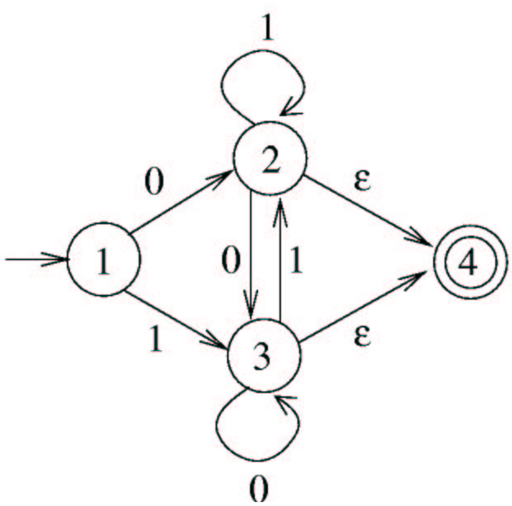
4. (20 points) Regular Expressions and GNFAs

a. Write a regular expression for the language L = {*w* | *w* starts with a 0 and has odd length} over the alphabet $\Sigma = \{0,1\}$.

$0(\Sigma\Sigma)^*$ or equivalently, $0(00 \cup 01 \cup 10 \cup 11)^*$

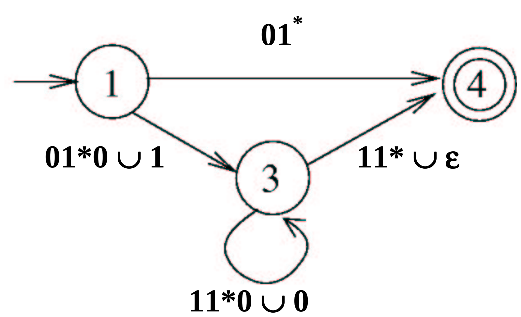
b. Using the construction from the text/lecture notes for converting a GNFA to a regular expression, remove state number 2 from the GNFA shown on the left. Show the result by labeling, with the appropriate regular expressions, the 4 resulting transition arrows between states in the GNFA on the right. Note: The special start and accept states have already been added (states 1 and 4).

Before



After removing state 2

(Label the 4 transition arrows)



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5. (20 points) Pumping Lemma and Non-Regular Languages

Let Σ be the set containing "(" and ")" i.e. $\Sigma = \{ (,) \}$. Define the language L_B as the set of all strings over Σ consisting of balanced parentheses. L_B can be defined recursively as follows: a string *w* is in L_B iff (1) *w* is the empty string, or (2) *w* is of the form: (w_1) for some w_1 in L_B, or (3) *w* is of the form: w_1w_2 for some w_1 and w_2 in L_B.
Examples of strings in L_B: (), (()), (((()))), ()(), (())(), (()(()))
Examples of strings not in L_B: (, ((()), ())), (()()(, ((())(((equivalent to $(^3)^2(^2$!)

Using the pumping lemma, prove that L_B is not a regular language.

1. **Assume that L_B is regular.**
2. **This implies that it can be "pumped." Let the pumping length be p.**
3. **Consider the string $s=(^p)^p$. Note that $|s| = 2p \geq p$**
4. **Let x,y,z be any three strings such that xyz=s, with $|xy| \leq p$ and $|y| \geq 1$. Note that by our choice of s, $y=(^k$ for some $k \geq 1$.**
5. **Choose i = 0 (other choices, e.g. i = 2, will also work). Then, $xy^iz = xz = (^{p-k})^p$ for some $k \geq 1$. xz is clearly not in L_B. This contradicts the pumping lemma. Therefore, L_B is not regular.**