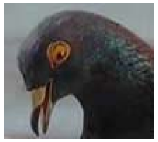
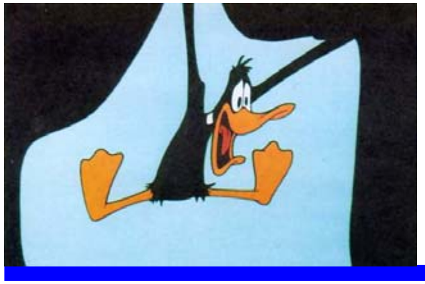
Review of Proof Techniques

- **Contents of the CSE 322 Proofs Toolbox**:
  - **Proof by counterexample**: Give an example that disproves the given statement.
  - **Proof by contradiction**: Assume statement is false and show that it leads to a contradiction.
  - **Proof by construction**
  - **Proof of set equality** A = B: Show $A \subseteq B$ and $B \subseteq A$
  - **Proof of "X iff Y"** (or $X \Leftrightarrow Y$) statements
  - **Proof by induction**
  - Avian technique #1: **Pigeonhole principle**
  - Avian technique #2: **Dovetailing**
  - CS Theoretician's favorite: **Diagonalization**





Proof Techniques Review: The Big picture

- **Proof by contradiction**: Assume statement is false and show that it leads to a contradiction
  - E.g.: Prove: Complement of any finite subset of Z is infinite
- **Proof by construction**: Show that a statement can be satisfied by constructing an object using what is given
  - E.g.: Show that for all c, $\exists\, n_0$ s.t. $n^2 > cn$ for all $n \geq n_0$
- **Proof of set equality** A = B: Show $A \subseteq B$ and $B \subseteq A$
  - E.g.: De Morgan's Law (one of two):
    $A - (B \cup C) = (A - B) \cap (A - C)$
- **Proving "X iff Y" statements**: Prove $X \Rightarrow Y$ ("X only if Y") and $Y \Rightarrow X$ ("X if Y")
  - E.g.: For all real numbers x, show $\lfloor x \rfloor = \lceil x \rceil$ iff $x \in Z$

Review: Avian Technique #1

- **Pigeonhole principle**: If A and B are finite sets and |A| > |B|, then there is no one-to-one function from A to B
  - f : A→B is one-to-one if for any distinct x, y ∈ A, f(x) ≠ f(y)
  - Idea: "more pigeons than pigeonholes" → 2 pigeons are shackin' up (at least one pigeonhole contains two pigeons)
  - E.g. In a room of 13 or more people, at least 2 have same birthmonth
  - Proof? By induction on |B|
- What is "Proof by Induction"?

Proof by Induction

- **Proof by induction** (very common in CS Theory): 2 steps –
  1. Basis Step: Show statement is true for some finite value n_0, typically $n_0 = 0$ or 1

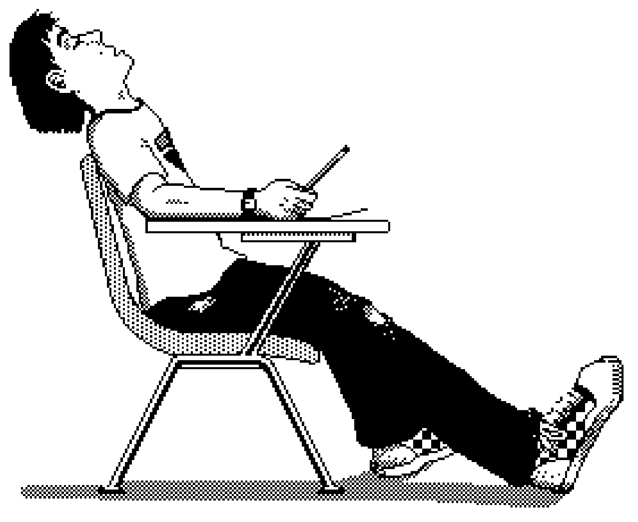
  2. Induction Hypothesis and Induction Step: Assume statement is true for some fixed but arbitrary $k \geq n_0$. Show it is also true for $k + 1$

  - Example: Show that for all $n \geq 1$, $1 + 2 + \ldots + n = n(n+1)/2$

To Infinity and Beyond (with apologies to Disney)

- Sizing up sets: Cardinality of a set and countably infinite sets
- Avian Technique #2 – **Dovetailing**: Useful for showing union of any finite or countably infinite collection of countably infinite sets is again countably infinite
  - Set A is *countably infinite* if there is a 1-1 correspondence ("bijection") between N (the set of natural numbers) and A
  - E.g. Use dovetailing to show Z and $N \times N$ are both countably infinite
  - A set is uncountable if it is neither finite nor countably infinite
- Diagonalization and Uncountable Sets: See <u>pages 174-178</u> in the text for a nice introduction and more examples.
  - E.g.: Set of real numbers in the interval (0,1) is uncountable
- See Handout #1 for more details…

Are we done with this review yet?



Enter…the finite automaton…