CSE 322: Regular Expressions and Finite Automata

- **Definition of a <u>Regular Expression</u>**
  - R is a regular expression iff
    R is a string over $\Sigma \cup \{ \varepsilon, \emptyset, (,), \cup, * \}$ and R is:
    1. Some symbol $a \in \Sigma$, <u>or</u>
    2. ε, <u>or</u>
    3. $\emptyset$, <u>or</u>
    4. $(R_1 \cup R_2)$ where R_1 and R_2 are regular exps., <u>or</u>
    5. $R_1R_2 = R_1 \circ R_2$ where R_1 and R_2 are reg. exps., <u>or</u>
    6. $R_1{}^*$ where R_1 is a regular expression.

- **Precedence**: Evaluate * first, then $^{\circ}$, then $\cup$
  - E.g. $0 \cup 11^* = 0 \cup (1^\circ (1^*)) = \{0\} \cup \{1, 11, 111, \ldots\}$

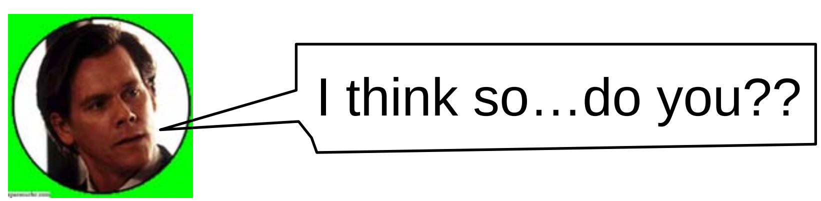
Examples

- What is R for each of the following languages?
  1. L(R) = {w | w contains exactly two 0's}
  2. L(R) = {w | w contains at least two 0's}
  3. L(R) = {w | w contains an even number of 0's}
  4. L(R) = {w | w does not contain 00}
  5. L(R) = {w | w is a valid identifier in C} (or in Java)
  6. L(R) = {w | w is a word heard on the MTV show "The Osbournes"}

Are u saying our
language is regular??

Regular Expressions and Finite Automata

- What is the relationship between regular expressions and DFAs/NFAs?
- Specifically:
  1. **R → NFA**? Given a reg. exp. R, can we create an NFA N such that L(R) = L(N)?
  2. **NFA → R**? Given an NFA N (or its equivalent DFA M), can we come up with a reg. exp. R such that L(M) = L(R)?



From Regular Expressions to NFAs

- Problem: Given *any* regular expression R, how do we construct an NFA N such that L(N) = L(R)?
- Soln.: Use the multi-part definition of regular expressions!!
  - Show how to construct an NFA for each possible case in the definition: R = *a*, or R = ε, or R = ∅, or R = (R1 ∪ R2), or R = R1°R2, or R = R1*.

Told ya ‘twas possible!

- Example: Draw NFA for 10Σ*01

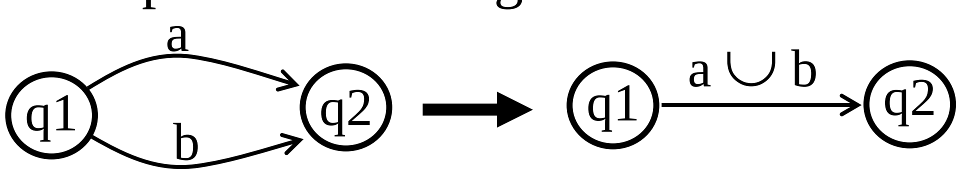
From NFAs/DFAs to Regular Expressions

- Problem: Given *any* NFA (or DFA) N, how do we construct a regular expression R such that L(N) = L(R)?

- Solution:
  - **Idea**: Collapse 2 or more edges in N labeled with single symbols to a *new edge* labeled with an *equivalent regular expression*
  - This results in a **"generalized" NFA** (**GNFA**)
  - **Our goal**: Get a GNFA with 2 states (start and accept) connected by a single edge labeled with the required regular expression R

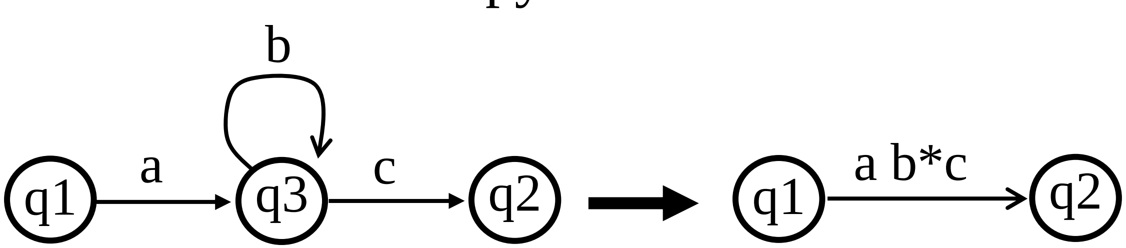
From NFAs/DFAs to Regular Expressions

- Steps for extracting regular expressions from NFAs/DFAs:
  1. Add new start state connected to old one via an ε–transition
  2. Add new accept state receiving ε–transitions from all old ones
  3. Keep applying 2 rules until only start and accept states remain:
     1. Collapse Parallel Edges:



Note: Also applies when q1 = q2

     2. Remove "loopy" states:



Note: Also applies when q1 = q2

(Example: On board and in textbook)

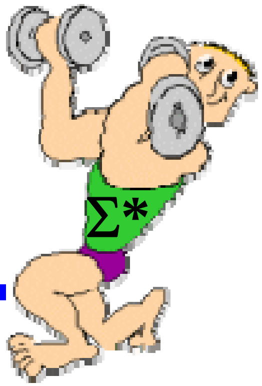
Beyond the Regular world…

- Are there languages that are *not* regular?
  - How do we prove it?

- **Idea:** If a language violates a property obeyed by all regular languages, it cannot be regular!
  - **Pumping Lemma** for showing *non-regularity* of languages

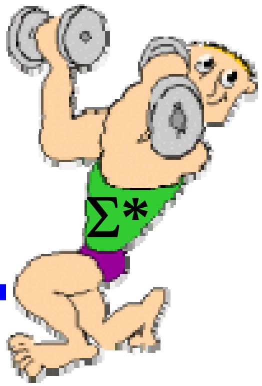
http://www.ipjnet.com/schwarzenegger2/pages/arnold_01.htm

The Pumping Lemma for Regular Languages



- **What is it?**
  - A statement ("lemma") that is true for all regular languages
- **Why is it useful?**
  - Can be used to show that certain languages are *not regular*
  - How? *By contradiction*: Assume the given language is regular and show that it does not satisfy the pumping lemma

More about the Pumping Lemma



- **What is the idea behind it?**
  - Any regular language L has a DFA M that recognizes it
  - If M has **p states** and accepts a string of length ≥ p, the sequence of states M goes through must contain a **cycle** (repetition of a state)
  - Why?
    - Due to the *pigeonhole principle*! p states allow at most p-1 transitions before a state is repeated.
  - Therefore, *all strings* that make M go through this cycle 0 or any number of times are also accepted by M and *should be in L*.

Formal Statement of the Pumping Lemma

- **Pumping Lemma**: If L is regular, then $\exists$ p such that $\forall$ s in L with $|s| \geq$ p, $\exists$ x, y, z with $s = xyz$ and:
  1. $|y| \geq 1$, and
  2. $|xy| \leq$ p, and
  3. $xy^iz \in L \ \forall\ i \geq 0$

- Proof on board…(also in the textbook)

- Proved in 1961 by Bar-Hillel, Peries and Shamir

Pumping Lemma in Plain English

- Let L be a regular language and let p = "pumping length" = no. of states of a DFA accepting L
- Then, any string *s* in L of length ≥ p can be expressed as *s* = *xyz* where:
  - *y* is not empty (*y* is the cycle)
  - $|xy| \leq p$ (cycle occurs within p state transitions), and
  - any "pumped" string xy^iz is also in L for all $i \geq 0$ (go through the cycle 0 or more times)

Using The Pumping Lemma

- In-Class Examples: Using the pumping lemma to show a language L is *not regular*
  - 5 steps for a proof by contradiction:

1. Assume L is regular.
2. Let p be the pumping length given by the pumping lemma.
3. Choose cleverly an *s* in L of length at least p, such that
4. For *all ways* of decomposing *s* into *xyz*, where $|xy| \leq p$ and *y* is not null,
5. There is an $i \geq 0$ such that xy^iz is not in L.

Proving Non-Regularity using the Pumping Lemma

- In class examples: Show the following are not regular
  - $L_1 = \{0^n1^n \mid n \geq 0\}$ over the alphabet $\{0, 1\}$
  - $L_2 = \{ww \mid w \text{ in } \{0, 1\}^*\}$